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A Low-Complexity Permutation-Based PTS Scheme for PAPR Reduction in OFDM Systems

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Abstract—**The high peak-to-average power ratio (PAPR) of the transmitted signal is one of the main drawbacks of orthogonal frequency division multiplexing (OFDM) systems. Partial transmit sequences (PTS) is a non-distortion technique to reduce the PAPR of OFDM systems, where the input data block is partitioned into some disjoint subblocks and then combined optimally to form a low-PAPR output signal. The conventional PTS scheme has a good PAPR reduction performance, but it usually involves high computational complexity. In this paper, we propose a new permutation-based PTS (PB-PTS) scheme is for OFDM systems, where the nonzero elements of each subblock are permutated and weighted to form more candidate signals for selection. Also, we generate cost functions by adding the sample powers to select samples to estimate the peak power of candidate signals and use the shift property of inverse fast Fourier transform to further reduce the computational complexity of the PB-PTS scheme. Computer simulations show that the proposed PB-PTS scheme achieves a better PAPR reduction performance than the conventional PTS scheme, but has much lower computational complexity.**

***Index Terms*—inverse fast Fourier transform (IFFT), orthogonal frequency division multiplexing (OFDM), partial transmit sequences (PTS), peak-to-average power ratio (PAPR).**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive multicarrier modulation technique that has high spectral efficiency and is robust to the frequency selective fading channels [1]. It is widely used in modern wireless communication systems, such as wireless local area networks (WLANs) [2], wireless metropolitan area networks (WMANs) [3], and 3GPP Long Term Evolution (LTE) systems [4]. One main drawback of OFDM systems is that the transmitted signal may have a high peak-to-average power ratio (PAPR). A high-PAPR OFDM signal may degrade the efficiency of power amplifiers and cause undesired signal distortion.

Numerous techniques have been developed to reduce the PAPR of OFDM systems [5]. Partial transmit sequences (PTS) is a non-distortion technique [6]. In the conventional PTS (CPTS) scheme, the input data block is partitioned into several disjoint subblocks. The inverse fast Fourier transform (IFFT) of each subblock is multiplied by a set of rotation factors and then combined to form various candidate signals, where the one with the lowest PAPR is selected for transmission. The CPTS scheme significantly reduces the PAPR of OFDM signals, but it has high computation complexity to find the optimal candidate signal. Many techniques were proposed to simplify the optimization process in the CPTS scheme [7]-[10], such as reducing the number of candidate signals [7] or the number of samples used to estimate the peak power of the candidate signals [8], [9].

In this paper, a new permutation-based PTS (PB-PTS) scheme is proposed to reduce the PAPR of OFDM systems. In the PB-PTS scheme, the nonzero elements of each subblock are permutated, which is equivalent to generate alternative forms of the input sequence. Then the permutated subblocks are combined optimally to form a low-PAPR signal. Also, an efficient algorithm is developed to reduce the computational complexity for the proposed PB-PTS scheme. Simulations show that the PB-PTS scheme achieves a better PAPR reduction performance than the CPTS scheme, but has lower computational complexity.

II. BACKGROUND

A. PAPR of OFDM Systems

In an N-subcarrier system, the discrete-time OFDM signal of the input data block $\mathbf{X} = [X_0, X_1, ..., X_{N-1}]^T$ is expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}}, \; n = 0,1,\ldots,N-1 \qquad (1)$$

where X_k denotes the data symbol modulated by the kth subcarrier. Let $\mathbf{x} = [x_0, x_1, ..., x_{N-1}]^T$ be the vector of the discrete-time OFDM signal. The PAPR of the signal x_n in (1) is defined as the ratio of the peak power to the average power of x_n, expressed as

$$\text{PAPR} = \frac{\max\limits_{0 \le n \le N-1} |x_n|^2}{E\left\{|x_n|^2\right\}} \qquad (2)$$

where $E\{\cdot\}$ denotes the expected value.

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B. Conventional PTS (CPTS) Scheme

In the CPTS scheme, the input data block **X** is partitioned into *M* disjointed subblocks first, denoted as

$$\mathbf{X}_m=[X_{m,0},\ X_{m,1},\ \ldots,\ X_{m,N-1}]^T,\ 1\le m\le M \tag{3}$$

where only *N*/*M* elements are nonzero in each subblock. The IFFT of subblock $\mathbf{X}_m$, i.e., $\mathbf{x}_m=[x_{m,0},\ x_{m,1},\ \ldots,\ x_{m,N-1}]^T$, is multiplied by a rotation factor $b_m^c=e^{j\theta_i}$ with $\theta_i\in[0,\ 2\pi)$ and then combined to form the *c*th candidate signal $\mathbf{x}^c$, expressed as

$$\mathbf{x}^c=\sum_{m=1}^{M}b_m^c\mathbf{x}_m=[x_0^c,\ x_1^c,\ \ldots,\ x_{N-1}^c]^T,\ 1\le c\le C \tag{4}$$

where *C* is the number of candidate signals. The candidate signal with the lowest PAPR among the *C* signals is transmitted. Normally, b_1^c is fixed without degrading the PAPR reduction performance of the CPTS scheme.

The CPTS scheme has a good PAPR reduction performance, but it has high computational complexity to find the optimal output signal. Also, it needs to send side information to the receiver for indicating the used vector of rotation factors $\mathbf{b}^c=[b_1^c,\ b_2^c,\ \ldots,\ b_M^c]^T$ of the transmitted signal. In the next section, we will develop a new PTS method based on the permutation of subblocks.

III. LOW-COMPLEXITY PERMUTATION-BASED PTS SCHEME

A. Subblock Permutation in PTS

Assume that the adjacent subblock partition method is used to divide the input data block **X** into *M* subblocks evenly in the CPTS scheme. Let $\tilde{\mathbf{X}}_m$ represent the *N*/*M* nonzero elements in subblock $\mathbf{X}_m$. Thus, **X** can be represented as

$$\mathbf{X}=[\tilde{\mathbf{X}}_1,\ \tilde{\mathbf{X}}_2,\ \ldots,\tilde{\mathbf{X}}_M]^T. \tag{5}$$

In the CPTS scheme with *M* subblocks, there are at most W^{M-1} candidate signals generated if the rotation factor b_m^c with $2\le m\le M$ has *W* various values. If we permute the nonzero elements $\tilde{\mathbf{X}}_m$ for $2\le m\le M$ in (5), more candidate signals will be formed to improve the PAPR reduction performance for the CPTS scheme. During the permutation process, the position of $\tilde{\mathbf{X}}_1$ is fixed and the other $M-1$ items in (5) have $(M-1)!$ permutations. Thus, $(M-1)!\times W^{M-1}$ candidate signals can be generated totally in the permutation process. Fig. 1 shows an example of subblock permutation for the *M*= 4 case. However, more candidate signals need more computational complexity.

B. Reduction of Computational Complexity

The increment of computational complexity in the proposed permutation-based PTS (PB-PTS) scheme is to find the IFFT of the permutated subblocks and the process to find the optimal transmitted signal among the candidate signals. For the first problem, we use the shift property of IFFT to simplify the computational complexity. Let $\mathbf{X}_m^{f_m}$ denote the subblock $\mathbf{X}_m$ in which the elements have a circular shift f_m in the frequency domain and $\mathbf{x}_m^{f_m}=$ $IFFT\{\mathbf{X}_m^{f_m}\}=[x_{m,0}^{f_m},\ x_{m,1}^{f_m},\ \ldots,\ x_{m,N-1}^{f_m}]^T$, where

$$\begin{aligned}
x_{m,n}^{f_m}&=\frac{1}{N}\sum_{k=0}^{N-1}X_{m,k}e^{\frac{j2\pi(k-f_m)n}{N}}\\
&=\frac{1}{N}e^{\frac{-j2\pi f_m n}{N}}\sum_{k=0}^{N-1}X_{m,k}e^{\frac{j2\pi kn}{N}}\\
&=x_{m,n}e^{\frac{-j2\pi f_m n}{N}}\\
&=h_{m,n}^{f_m}x_{m,n}
\end{aligned} \tag{6}$$

with $h_{m,n}^{f_m}=e^{\frac{-j2\pi f_m n}{N}}$ and $n=$ 0, 1, ..., $N-1$. Let $\mathbf{H}_m^{f_m}=$ $[h_{m,0}^{f_m},\ h_{m,1}^{f_m},\ \ldots,\ h_{m,N-1}^{f_m}]^T$. Then

$$\mathbf{x}_m^{f_m}=\mathbf{H}_m^{f_m}\otimes\mathbf{X}_m \tag{7}$$

where $\otimes$ denotes the element-wise multiplication. If f_m is a multiple of *N*/4, then the elements of $\mathbf{H}_m^{f_m}$ will be in the set {±1, ±*j*} and the multiplications in (7) can be ignored. For example, if f_m is *N*/4, then $\mathbf{H}_m^{f_m}=[1,\ -j,\ -1,\ j,\ 1,\ \ldots,\ 1,\ -j,\ -1,\ j]^T$. Therefore, we can use (7) to replace the IFFT operation to find the time-domain signal $\mathbf{x}_m^{f_m}$ for the permutated subblock $\mathbf{X}_m^{f_m}$.

An efficient method has been developed in [9] to reduce the number of samples used to estimate the peak power of each candidate signal for the PTS technique, where the cost function of the sample located at time *n* is defined as

$$Q_n=\sum_{m=1}^{M}|x_{m,n}|^2,\ n=0,1,\ldots,N-1. \tag{8}$$

Only the samples with Q_n greater than a predefined threshold are used to estimate the peak power of each candidate signal, so the computational complexity is reduced.

Assume that the subblocks still remain disjointed after permutation in the proposed PB-PTS scheme. Then the cost function for the PB-PTS scheme is defined as

$$\begin{aligned}
Q_n&=\sum_{m=1}^{M}|x_{m,n}^{f_m}|^2=\sum_{m=1}^{M}|h_{m,n}^{f_m}x_{m,n}|^2\\
&=\sum_{m=1}^{M}|h_{m,n}^{f_m}|^2\times|x_{m,n}|^2=\sum_{m=1}^{M}|x_{m,n}|^2
\end{aligned} \tag{9}$$

which is the same as that in (8). Thus, the sample locations selected by using the cost function in (8) are valid for the candidate signals formed by the permutated subblocks in the PB-PTS scheme. Also, the CPTS method can be regarded as the case of the proposed PB-PTS scheme with the frequency-shift $f_m = 1$ for all subblocks. Fig. 2 shows the block diagram of the proposed PB-PTS scheme.

C. Analysis of Computational Complexity

Both of the CPTS and the PB-PTS schemes with M subblocks needs M IFFT units, so only the computational complexity of the optimization process is discussed. Assume that the rotation factor b_1^c is 1 for both methods. The CPTS scheme with C candidate signals needs $CN(M–1)$ complex multiplications and $CN(M–1)$ complex additions in (4); CN complex multiplications to compute the sample powers and $CN–1$ comparisons to find the optimal signal among the C candidate signals.

In the proposed PB-PTS scheme, only the samples with cost functions greater than a threshold α_N are used to estimate the peak power during the optimization process. The minimum possible peak power in the OFDM system with N subcarriers is defined as

$$\Phi_N^{\gamma} = -\sigma^2 \ln[\frac{\ln(1-\gamma)}{-N(\frac{\pi}{3}\ln N)^{0.5}}] \tag{10}$$

where σ^2 is the mean power and γ is the lower bound of the probability that the peak power is greater than Φ_N^{γ} [9]. For example, if $\gamma = 0.9999$, it means that 99.99% of the peak power in the OFDM system with N subcarriers is not less than $\Phi_N^{\gamma=0.9999}$. In [9], the authors also induced that the probability to estimate the peak power correctly is not less than γ when $\alpha_N = \Phi_N^{\gamma} / M$ is the threshold of the cost function Q_n used to select the samples. If p_α is the probability that Q_n is higher than α_N, then the number of samples used to estimate the peak power is $p_\alpha N$. The probability distribution of p_α was also derived in [9].

For the proposed PB-PTS scheme, it needs MN complex multiplications and $(M–1)N$ real additions to generate the cost function in (8); N comparisons with the threshold α_N to select the samples. There are only $p_\alpha N$ samples in one candidate signal involved in the optimization. It needs $C_{\mathrm{PB}} p_\alpha N(M-1)$ complex multiplications and $C_{\mathrm{PB}} p_\alpha N(M-1)$ complex additions to form C_{PB} candidate signals; $C_{\mathrm{PB}} p_\alpha N$ complex multiplications to compute the sample powers and $C_{\mathrm{PB}} p_\alpha N - 1$ comparisons to find the optimal signal with the minimum peak power among the C_{PB} candidate signals. The comparisons of computational complexity are summarized in Table I, in which L times oversampling is considered so the term N is replaced by LN. Note that a complex addition is equivalent to two real additions and a comparison is equivalent to a real addition.

VI. Simulation Results

Computer simulations are performed to compare the PAPR reduction performance for the proposed PB-PTS and the CPTS scheme. The OFDM system is assumed to have N= 1024 subcarriers with 16-QAM modulation and the mean power $\sigma^2 = 1$. L = 4 times oversampling technique is used to obtain the discrete-time OFDM signal. The adjacent subblock partition technique is used in both CPTS and PB-PTS schemes to partition the input data block **X** into M= 4 subblocks. The elements in the vector of rotation factors $\mathbf{b}^c = [1, b_2^c, b_3^c, ..., b_M^c]^T$ are selected from the set $\{\pm 1, \pm j\}$ to simplify the multiplication computations in (4). The number of candidate signals is C= 64 for the CPTS scheme. In the PB-PTS scheme, the first subblock is not permutated, so the maximum number of candidate signals is 384.

A. Comparison of PAPR Reduction Performance

Fig. 3 shows the complementary cumulative distribution function (CCDF) of PAPR for both schemes, where the proposed PB-PTS scheme does not use the cost functions and the number of candidate signals is from 64 to 384. It shows that the PB-PTS scheme has a better PAPR reduction performance than the CPTS scheme when it has more than 64 candidate signals and the improvement of PAPR reduction performance is not significant when the number of candidate signals is greater than 320.

Fig. 4 shows the PAPR reduction performance for the proposed PB-PTS scheme that uses the cost functions in (8) to estimate the peak power of each candidate signal, where $\Phi_{N=1024}^{\gamma=0.9999} = 5.71$ obtained by using (10) and the threshold $\alpha_N = \beta \times \Phi_{N=1024}^{\gamma=0.9999}$ with β = 0.25, 0.33, and 0.4. Fig. 4 shows that, when β is not greater than 0.33, the PB-PTS scheme has almost the same PAPR reduction performance as the case that uses all samples to estimate the peak power for both C_{PB} = 64 and 384 cases.

B. Comparison of Computational Complexity

The proposed PB-PTS scheme uses part of the samples to estimate the peak power of each candidate signal. From the results shown in Figs. 3 and 4, we choose the case with C_{PB} = 320 and β = 0.33 for the PB-PTS scheme, where the threshold $\alpha_N = 0.33 \times 5.71 = 1.88$ and $p_\alpha = 0.058$. By using Table I, we obtain that, when C_{PB} = 320, the proposed PB-PTS scheme achieves a better PAPR reduction performance than the CPTS scheme, but with only about 31% multiplications and 30% additions of that of the CPTS scheme.

V. Conclusion

In this paper, a low-complexity PB-PTS scheme has been proposed for PAPR reduction in OFDM systems. In the proposed PB-PTS scheme, we use a subblock permutation method to increase the number of candidate signals. In addition, a cost function is used to select the samples of each candidate signal for the peak power estimation in the PB-PTS scheme. Computer simulations show that the proposed PB-PTS scheme achieves a better PAPR reduction performance than the CPTS scheme, but with much lower computational complexity.

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TABLE I
ANALYSIS OF COMPUTATIONAL COMPLEXITY FOR THE PROPOSED PB-PTS AND THE CPTS SCHEMES

	Complex ‘×’	Real ‘+’
PB-PTS	$LNM +$ $C_{\mathrm{PB}} p_{\alpha} LN(M-1)^{\dagger} +$ $C_{\mathrm{PB}} p_{\alpha} LN$	$LN(M-1)+LN+$ $2C_{\mathrm{PB}} p_{\alpha} LN(M-1) +$ $C_{\mathrm{PB}} p_{\alpha} LN - 1$
CPTS	$CLN(M-1)^{\dagger}+CLN$	$2CLN(M-1)+CLN-1$

†These terms can be ignored if the rotation factors are in the set $\{\pm 1, \pm j\}$.

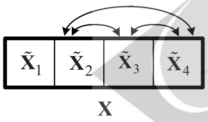

Fig. 1 Example of subblock permutation, where the input data block **X** is partitioned into four subblocks and the first subblock is fixed.

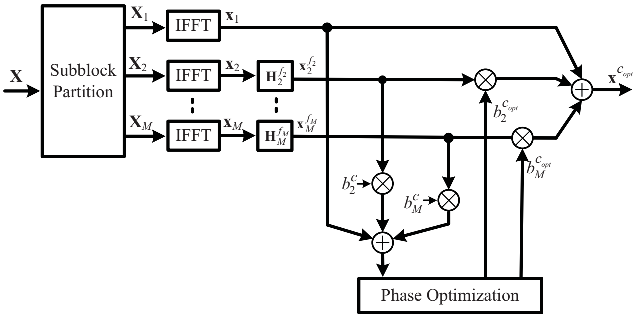

Fig. 2 Block diagram of the proposed PB-PTS scheme.

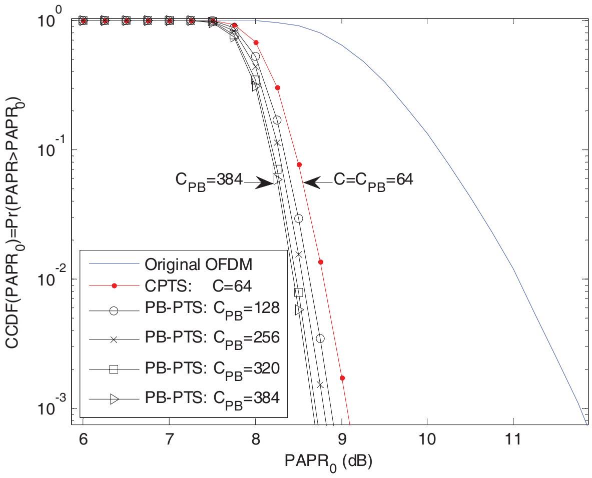

Fig. 3 Comparison of PAPR reduction performance for the CPTS scheme and the proposed PB-PTS scheme with M= 4, where the number candidate signals for the latter is C_{PB} = 64, 128, 256, 320, and 384.

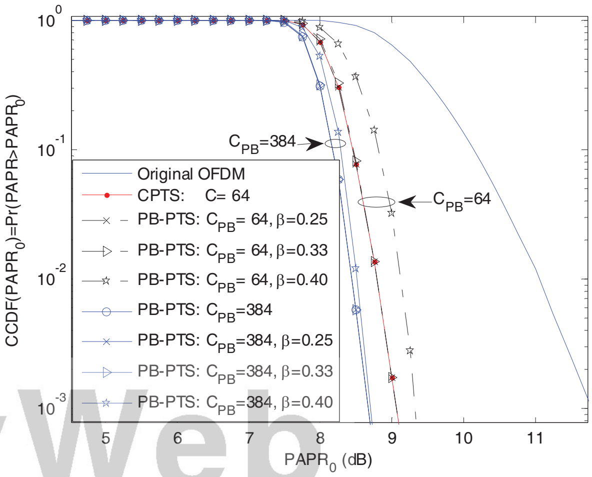

Fig. 4 Comparison of PAPR reduction performance for the CPTS scheme and the proposed PB-PTS scheme with M= 4, where the threshold $\alpha_N = \beta \times \Phi_{N=1028}^{\gamma=0.9999}$ with $\beta = 0.25, 0.33$, and 0.4 is used for the latter.