S-parameter Measurement of an N-port Reciprocal Network Using One-port Vector Network Analyzer

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***Abstract* —This paper describes a scattering matrix reconstruction method with simulation results. In this method, one can obtain the scattering matrix of an n-port reciprocal network through a set of one-port measurements using a vector network analyzer. Among these measurements, changing of the measured port of the device under test is not necessary. However, the off-diagonal elements of the reconstructed scattering matrix may have ambiguity in its phase information, and this is the cost of only using one-port measurements.[1]**

I. Introduction

The scattering matrix of a multi-port network could be obtained either from the direct full port measurement, or from reconstruction methods which use the data obtained from reduced port measurements [1]-[4]. Although the former is a straight forward approach to get the result, the necessary instrument is usually costly and sometimes even impracticable. Besides, the calibration of a multi-port network analyzer should be treated carefully [5]-[7] due to the large number of error terms and the complicated error path [6].

Unlike the full port measurement, the requirement for instrument of reduced port method is easier. There are also well developed calibration methods could be used for those methods which can reduce the measured ports to two [1], [3], [4]. However, there are three problems would affect the accuracy. The first one is the numerical issue due to the algorithm used to reconstruct the original scattering matrix [1], [3], [8]. The second one is the numerous reconnections during those necessary measurements. The last one is the confidence in those terminations connected on the unmeasured ports, because most of the reduced port methods need complete information about these terminations.

This paper is attempted to reduce the measured port to one. In this study, we will find that one can acquire the scattering matrix (which will lose the exact phase information of the off-diagonal elements) of an n-port reciprocal network through multiple one-port measurements without changing the measured port of the device under test. The formulation is described in Sec. II and the simulation results are given in Sec. III.

[1] This work was supported in part by the NSC under Grants 100-2221-E-002-225-MY3 and 101-2221-E-002-128-MY2.

II. Formulation

In this section, we will describe the formulation of this method by starting from one-port measurements to reconstruct the two-port scattering matrix of an n-port reciprocal network. Its diagonal elements would be used to reconstruct the three-port scattering matrix. Similarly, the four-port scattering matrix (if it exists) could be found by using the diagonal elements of the three-port scattering matrix. This procedure would be repeated until the full port scattering matrix is found.

Taking a two-port circuit into consideration for the time being, if its port 2 is terminated with Γ_2, the reflect coefficient at port 1 is

$$S_{11}^{(2)} = S_{11} + \frac{S_{12}S_{21}\Gamma_2}{1 - S_{22}\Gamma_2} \tag{1}$$

or

$$S_{11} + S_{11}^{(2)}\Gamma_2 S_{22} + \Gamma_2(S_{12}S_{21} - S_{11}S_{22}) = S_{11}^{(2)}. \tag{2}$$

If we regard the term of $S_{12}S_{21}$-$S_{11}S_{22}$ in (2) as a single unknown, (2) becomes a linear equation with three unknowns. It means with three different terminators Γ_{2a}, Γ_{2b}, and Γ_{2c}, S_{11} and S_{22} would be solved. By substituting them into (2), $S_{12}S_{21}$ can be found. However, as mentioned above, only the diagonal elements would be used to reconstruct the three-port scattering matrix in the next step, we can leave S_{11} and S_{22} by eliminating $S_{12}S_{21}$ in (2). The resulted equations are actually a special case of (5) and (6) in [1], which are given as

$$\left(\frac{1}{\Gamma_{2a}} - \frac{1}{\Gamma_{2b}}\right)S_{11} + (S_{11}^{(2a)} - S_{11}^{(2b)})S_{22} = \frac{S_{11}^{(2a)}}{\Gamma_{2a}} - \frac{S_{11}^{(2b)}}{\Gamma_{2b}} \tag{3}$$

$$\left(\frac{1}{\Gamma_{2a}} - \frac{1}{\Gamma_{2c}}\right)S_{11} + (S_{11}^{(2a)} - S_{11}^{(2c)})S_{22} = \frac{S_{11}^{(2a)}}{\Gamma_{2a}} - \frac{S_{11}^{(2c)}}{\Gamma_{2c}}. \tag{4}$$

With the help of (3) and (4), one can easily find S_{11} and S_{22}.

Now we discuss one step further to the three-port case. For a three-port circuit, S_{11} and S_{22} obtained from (1), or (3) and (4), are actually $S_{11}^{(3)}$ and $S_{22}^{(3)}$ because the unused port 3 would be terminated with Γ_3. Note, $S_{11}^{(2a)} \sim S_{11}^{(2c)}$ in (3) and (4) are actually $S_{11}^{(3,2a)} \sim S_{11}^{(3,2c)}$ to represent the terminated port 3.

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DOI:10.6159/IJSE.2014.(4-2).45

Take $S_{11}^{(3)}$ for example. By replacing all the 2's in (1) with 3's, we will find that the unknowns related to $S_{11}^{(3)}$ are S_{11}, S_{33}, and $S_{13}S_{31}$. Similarly, to solve these three unknowns, we need three different Γ_3, given by Γ_{3a}, Γ_{3b}, and Γ_{3c}. This means one should terminate port 3 with Γ_{3a} and perform three one-port measurements on port 1 while Γ_{2a}, Γ_{2b}, and Γ_{2c} are alternately terminated on port 2. Replacing Γ_{3a} with Γ_{3b} and Γ_{3c}, and repeating the above process will help us to acquire $S_{11}^{(3a)} \sim S_{11}^{(3c)}$ and $S_{22}^{(3a)} \sim S_{22}^{(3c)}$ which are sufficient to solve S_{11}, S_{22}, S_{33}, $S_{13}S_{31}$, and $S_{23}S_{32}$. For a reciprocal three-port network, the off-diagonal elements are the square root of $S_{13}S_{31}$, and $S_{23}S_{32}$. Because the phase difference is 180° between the two roots, rough concept about phase information of the off-diagonal elements is helpful to choose the proper one.

So far, we have the entire three-port scattering matrix except for S_{12}. As (1) says, we need $S_{11}^{(2a)} \sim S_{11}^{(2c)}$ to solve it. Note that $S_{11}^{(2a)} \sim S_{11}^{(2c)}$ here are slightly different from those we used before, which are actually $S_{11}^{(3,2a)} \sim S_{11}^{(3,2c)}$ to represent the reflect coefficients with a perfectly matched port 3. To get $S_{11}^{(2a)}$, we need $S_{11}^{(3a,2a)}$, $S_{11}^{(3b,2a)}$, and $S_{11}^{(3c,2a)}$, which are all measured data at port 1. After $S_{11}^{(2b)}$ and $S_{11}^{(2c)}$ are found through the same procedure, the scattering parameter S_{12} is acquired. In addition, by choosing proper Γ's to reduce the condition number of (3) and (4), one may get a numerically stable result.

Table I shows the combination of the one-port measurements needed to reconstruct the scattering matrix of a three-port network. Specifically, there are 3^{n-1} measurements have to be performed for an n-port reciprocal network.

To summarize the reconstruction procedure, we can rewrite (1) in a more general form as

$$S_{11}^{(n,n-1,\ldots,3,2)} = S_{11}^{(n,n-1,\ldots,3)} + \frac{S_{12}^{(n,n-1,\ldots,3)} S_{21}^{(n,n-1,\ldots,3)} \Gamma_2}{1 - S_{22}^{(n,n-1,\ldots,3)} \Gamma_2}. \qquad (5)$$

From (5), it is clear that with three terminators placed in sequence, we can find some of the scattering parameters of the higher order in port number. A general form of (5) is shown below, which could help us to find the arbitrary scattering parameters of the next higher order in port number.

$$S_{ii}^{(n,n-1,\ldots,k-1,k,k+1,\ldots)} = S_{ii}^{(n,n-1,\ldots,k-1,k+1,\ldots)} + \frac{S_{ik}^{(n,n-1,\ldots,k-1,k+1,\ldots)} S_{ki}^{(n,n-1,\ldots,k-1,k+1,\ldots)} \Gamma_k}{1 - S_{kk}^{(n,n-1,\ldots,k-1,k+1,\ldots)} \Gamma_k}. \qquad (6)$$

Because all the necessary measurements to solve (6) could be found in the arrangement similar to Table I, it is possible to acquire the scattering matrix of an n-port reciprocal network using one-port vector network analyzer.

TABLE I
THE CONFIGURATION OF TERMINATORS FOR THREE-PORT CASE

Port 3	Γ_{3a}			Γ_{3b}			Γ_{3c}		
Port 2	Γ_{2a}	Γ_{2b}	Γ_{2c}	Γ_{2a}	Γ_{2b}	Γ_{2c}	Γ_{2a}	Γ_{2b}	Γ_{2c}

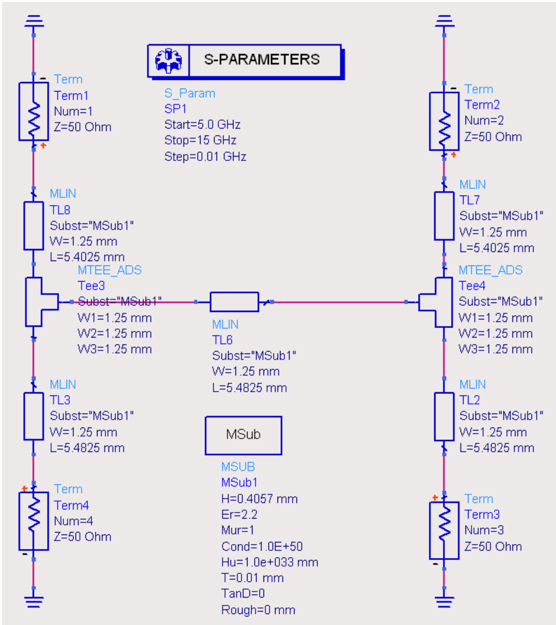


Fig. 1 The four-port microstrip circuit.

TABLE II
THE RESISTANT VALUES OF TERMINATORS

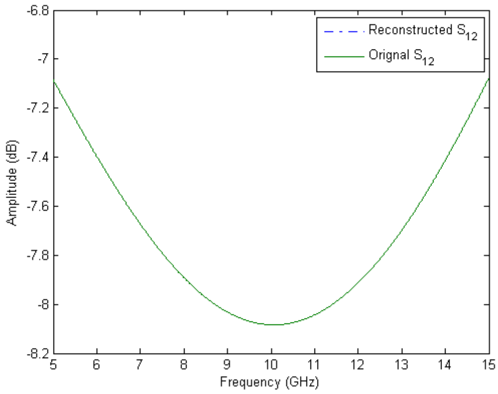
Port	Port 4			Port 3			Port 2		
Terminator	4a	4b	4c	3a	3b	3c	2a	2b	2c
Resistance(Ω)	65	75	80	35	30	25	70	20	85

III. SIMULATION RESULTS

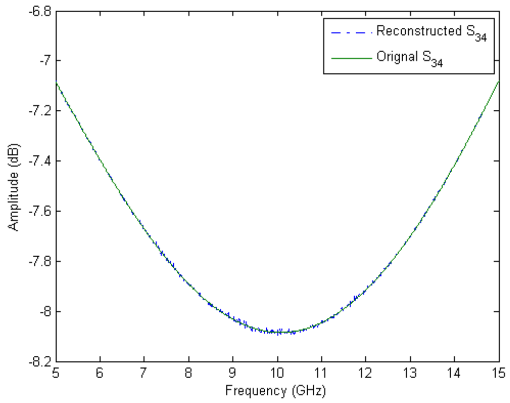
In this section, the simulation results are presented. It attempts to reconstruct a four-port H-shape reciprocal microstrip circuit, which is shown in Fig 1. The one-port measurements at port 1 are simulated by Agilent Advanced Design System 2004A [9] and the reconstruction is performed by MATLAB R2009a. Table II shows the resistance values of the terminators at ports 2, 3 and 4. The frequency range is from 5 GHz to 15 GHz with 1000 points.

Among all the reconstructed 16 scattering parameters, we display the simulation results of S_{12} and S_{34} because the former shows the best consistency while the latter has the largest error. These two results are shown in Fig. 2(a) and (b). A close examination of Fig. 2(b) is shown in Fig. 2(c). The reconstructed results are shown in close agreement with the original ones.

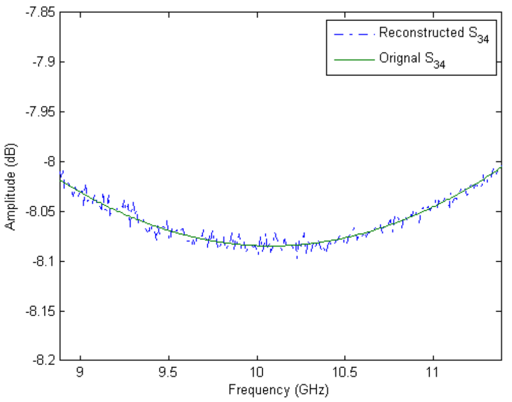
As for the other 14 reconstructed scattering parameters,



(a)



(b)



(c)

Fig. 2 Comparison of the original and reconstructed scattering parameters of (a) S_{12}, (b) S_{34} and (c) its closed examination.

we list the summation of errors in Table III. Note that because the phase could be directly found for the diagonal elements, the definitions of errors are slightly different between the diagonal and off-diagonal elements. For the diagonal elements, the error summation is defined as

$$100\% \times \sum_{f=f_{start}}^{f_{stop}} \frac{\left|S_{reconstructed}(f) - S_{original}(f)\right|}{\left|S_{original}(f)\right|}. \quad (7)$$

TABLE III

THE ERROR SUMMATION OF ALL 16 SCATTERING PARAMETERS

0.055375	0.054143	0.22434	0.082195
0.054143	4.3599	3.3442	22.4919
0.22434	3.3442	7.2178	36.8687
0.082195	22.4919	36.8687	2.8347

For the off-diagonal elements, the error summation is defined as

$$100\% \times \sum_{f=f_{start}}^{f_{stop}} \frac{\left\|S_{reconstructed}(f)\right| - \left|S_{original}(f)\right\|}{\left|S_{original}(f)\right|}. \quad (8)$$

IV. CONCLUSION

We have presented the method to obtain the scattering matrix of an n-port reciprocal network through one-port measurements. The network analyzer calibration is therefore simpler one-port calibration which has less requirements for calibration standards. Although the phase of the off-diagonal elements could not be directly solved from this method, a rough knowledge about the phase information is sufficient to choose the proper one because the two roots are 180° in difference.

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