Numerical Analysis of the Force Sensor Beam of the Chip

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Abstract— **The concept of chips originated in the late 1980s when many Euramerican research units realized that the development and application of chips, a comprehensive result of microelectronics, micromechanics, life science and biological message, would definitely bring about in the 21st century a biotechnology revolution. Chip research is still in its early stages of development, but there have been many great achievements thus far, such as the Gene Chip, DNA Chip or Microarray, Protein Chip, Micro-fluidics and multifunctional Lab-on-a-chip. The primary focus of this paper is the numerical analysis of force measurement on chips, which deduces the relation of the system through changing the spring constant, damping constant and Young's modulus of different materials. Finally, it puts forward a numerical example for calculation and explanation. All these results can serve as reference for future chip designs and improvements*.**

Index Terms— **force measurement, chip, numerical analysis, material**

I. INTRODUCTION

The problem of the free vibration of beams with concentrated mass has been discussed by several authors. Betzig et al. [1] studied the lateral displacement of a vibrating cantilever beam with a concentrated mass. Justine and Krishnan [2] obtained the matrix iteration for resilient support at one end. Chang [3] analyzed the vibration of a mass-loaded beam with a heavy tip mass by using the Laplace transform. Recently, Chang and his associate [4] adopted the same method to perform a vibration analysis of a beam with a two-degrees-of-freedom spring-mass system. Liu and Huang [5] studied a restrained cantilever beam with two concentrated masses. Chang [6] studied the eigenvalues of a viscously damped simple beam carrying point masses and springs. Haque and Saif [7] used MEMS force sensors for an in situ mechanical characterization of nano-scale thin films in SEM and TEM. Chang [8] investigated the sensitivity of an MEMS force sensor beam on the test chip.

In this paper, the dynamic responses of an MEMS force sensor beam have been considered. The natural frequency for force sensor beam vibration was derived, and an approximate solution was obtained using the Laplace transform method.

II. METHOD OF ANALYSIS

The partial differential equation of the bending vibrations of a force sensor beam with later spring, according to the Bernoulli-Euler theory [9], is the following well-known expression (Fig. 1):

$$EI(1+i\sigma)\frac{\partial^4 w(x,t)}{\partial x^4}+m\frac{\partial^2 w(x,t)}{\partial t^2}$$
$$+k_s\delta(x-x_s)w(x,t)+c_s\delta(x-x_s)\frac{\partial w(x,t)}{\partial t}=0 \qquad (1)$$

where E is the Young's modulus of the elasticity of the beam, σ is the hysteretic damping ratio, k_s is the spring constant, c_s is the damping constant, I is the area moment of inertia of the beam and $\delta(x)$ is the Dirac delta function.

Assume a solution of Eq. (1) as the form:

$$w(x,t)=W(x)e^{i\omega t}\,, \qquad (2)$$

Substituting Eq. (2) into Eq. (1) gives:

$$EI(1+i\sigma)W''''(x)-m\omega^2W(x)$$
$$+k_s\delta(x-x_s)W(x)+c_s(i\omega)\delta(x-x_s)W(x)=0 \qquad (3)$$

where ω is the natural frequency and $i=\sqrt{-1}$. The corresponding boundary conditions are:

$$W(0)=0\,,\ \frac{\partial W(0)}{\partial x}=0\,, \qquad (4)$$

$$W(L)=0\,,\ \frac{\partial W(L)}{\partial x}=0\,. \qquad (5)$$

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Take the Laplace transform of Eq. (3) in conjunction with the boundary conditions of Eqn. (4), and then apply the inverse Laplace transform to yield:

$$W(x) = \frac{\alpha}{2\beta^2}(\cosh ax - \cos ax)$$
$$+\frac{\gamma}{2a^3}(\sinh ax - \sin ax) - \frac{[k_s + c_s(i\omega)]W(x_s)}{2a^3 EI(1+i\sigma)}$$
$$[\sinh a(x-x_s) - \sin a(x-x_s)]\cdot H(x-x_s), \qquad (6)$$

$$a = \left(\frac{m\omega^2}{EI(1+i\sigma)}\right)^{\frac{1}{4}}. \qquad (7)$$

where $\alpha = W''(0)$, $\gamma = W'''(0)$ and $H(x)$ is the Heaviside unit step function. Then, substituting Eq. (5) into the boundary conditions of Eqn. (6) yields the following two equations:

$$\frac{1}{2a^2}\{(\cosh(aL)-\cos(aL)) -$$
$$\frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}[\sinh(a(L-x_s)) - \sin(a(L-x_s))]$$
$$\cdot(\cosh(ax_s) - \cos(ax_s))\}\alpha +$$
$$\frac{1}{2a^3}\{(\sinh(aL) - \sin(aL)) -$$
$$\frac{[k_s + c_s(i\omega)]}{2EIa^3(1+i\sigma)}[\sinh(a(L-x_s)) - \sin(a(L-x_s))]$$
$$\cdot(\sinh(ax_s) - \sin(ax_s))\}\gamma = 0, \qquad (7)$$

$$\frac{1}{2a}\{(\sinh(aL)+\sin(aL)) -$$
$$\frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}[\cosh(a(L-x_S)) - \cos(a(L-x_S))]$$
$$\cdot(\cosh(ax_s) - \cos(ax_s))\}\alpha +$$
$$\frac{1}{2a^2}\{(\cosh(aL) - \cos(aL)) -$$
$$\frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}[\cosh(a(L-x_s)) - \cos(a(L-x_s))]$$
$$\cdot(\sinh(ax_s) - \sin(ax_s))\}\gamma = 0. \qquad (8)$$

From the above equations, the characteristics equation can be found:

$$\frac{1}{2a^2}\{(\cosh(aL)-\cos(aL)) - \frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}$$
$$[\sinh(a(L-x_s)) - \sin(a(L-x_s))]$$
$$\cdot(\cosh(ax_s) - \cos(ax_s))\}\times$$
$$\frac{1}{2a^2}\{(\cosh(aL)-\cos(aL)) - \frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}$$
$$[\cosh(a(L-x_s)) - \cos(a(L-x_s))]$$
$$\cdot(\sinh(ax_s) - \sin(ax_s))\}\gamma$$
$$\frac{1}{2a}\{(\sinh(aL)+\sin(aL)) - \frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}$$
$$[\cosh(a(L-x_S)) - \cos(a(L-x_S))]$$
$$\cdot(\cosh(ax_s) - \cos(ax_s))\}$$
$$\frac{1}{2a^2}\{(\cosh(aL) - \cos(aL)) -$$
$$\frac{[k_s + c_s(i\omega)]}{2EI(1+i\sigma)a^3}[\cosh(a(L-x_s)) - \cos(a(L-x_s))]$$
$$\cdot(\sinh(ax_s) - \sin(ax_s))\}\gamma = 0. \qquad (9)$$

III. NUMERICAL RESULTS AND DISCUSSION

The system parameters used here were as follows: force sensor beam length $L = 500\mu m$; beam Young's modulus $E = 1.3\times10^{11} N/m^2$; beam density $\rho = 2.33\times10^3 kg/m^3$.

Equation (9) has an infinite number of roots, and the first roots were evaluated using the secant method.

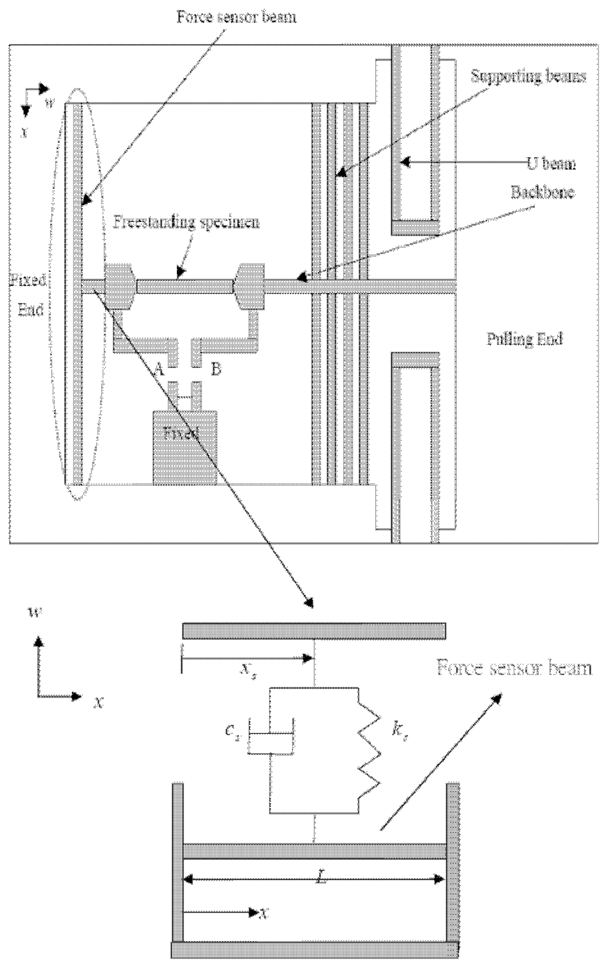

Figure1. The chip containing micro-electro-mechanical system force sensor beam.

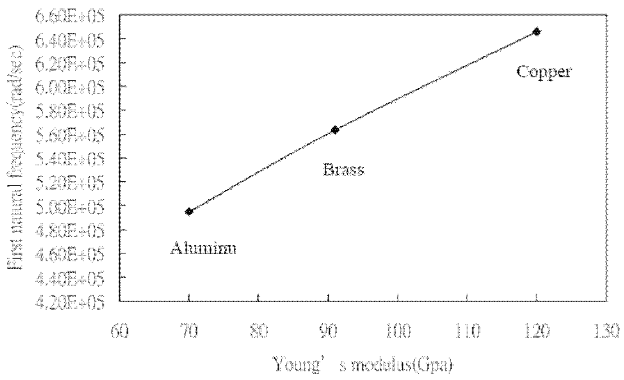

Figure 2. First natural frequency ω_1 versus Young's modulus E for $x_s = 0.5L$.

Figure 2 indicates the relationship between the first natural frequency and Young's modulus, from which it can be seen that when the Young's modulus of Aluminu is 70Gpa, the natural frequency of the test chip containing a micro-electro-mechanical system force sensor beam is 4.94971e+005rad/sec; if it is Brass, the natural frequency increases to 5.63318e+005rad/sec; if it is Copper, the natural frequency further increases to 6.45922e+005rad/sec. Combined with Eq (6), it was evident that the natural frequency was in proportion to Young's modulus. Hence, different materials result in different natural frequencies; the higher the Young modulus, the higher the natural frequency, which conforms to the vibration mechanics theory.

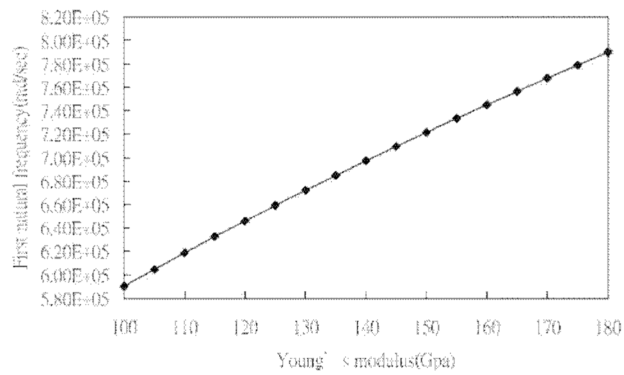

Figure 3. First natural frequency ω_1 versus Young's modulus E for $x_s = 0.5L$.

Figure 3 also indicates the relationship between the first natural frequency and Young's modulus, demonstrating that natural frequency increases with the increase of Young's modulus as in Fig. 2.

It can be observed from Table 1 that: (1) when the spring and the damping is placed where $x_s = 0$ m and $x_s = L$ m, there is no change in the hysteretic damping ratio, and the spring constant and damping constant are sufficient enough to alter the first natural frequency of the system to 6.69174e+005rad/sec; (2) when the spring and the damping are placed where $x_s = 0.5L, 0.75L$ m, the first natural frequency is in proportion to the spring constant, but in inverse proportion to the damping constant. The results obtained from the vibration mechanics theory and the above two points were reasonable and consistent with relevant theories.

Table 1 First natural frequency ω_1 of the force sensor beam on the chip

x_s (m)	σ	k_s (N/m)	c_s (N.sec/m)	ω_1 (rad/sec)
0	0	0	0	6.69174e+005
0	0	1	0	6.69174e+005
0	0	10	0	6.69174e+005
0	0	30	0	6.69174e+005
0	0	0	1e-5	6.69174e+005
0	0	0	1e-6	6.69174e+005
0	0	0	1e-7	6.69174e+005
0	1e-5	1	1e-5	6.69174e+005+3.34587e+000i
0.5L	0	0	0	6.69174e+005
0.5L	0	1	0	6.72944e+005
0.5L	0	10	0	7.07659e+005
0.5L	0	30	0	7.89113e+005
0.5L	0	0	1e-5	6.67822e+005+2.50959e+004i
0.5L	0	0	1e-6	6.69161e+005+2.51663e+003i
0.5L	0	0	1e-7	6.69174e+005+2.51670e+002i
0.5L	1e-5	1	1e-5	6.71578e+005+2.53627e+004i
0.75L	0	0	0	6.69174e+005
0.75L	0	1	0	6.70603e+005
0.75L	0	10	0	6.83520e+005
0.75L	0	30	0	7.12551e+005
0.75L	0	0	1e-5	6.69174e+005+3.34587e+000i
0.75L	0	0	1e-6	6.69174e+005+3.34587e-001i
0.75L	0	0	1e-7	6.69174e+005+3.34587e-002i
0.75L	1e-5	1	1e-5	6.70433e+005+9.58528e+003i
L	0	0	0	6.69174e+005
L	0	1	0	6.69174e+005
L	0	10	0	6.69174e+005
L	0	30	0	6.69174e+005
L	0	0	1e-5	6.69174e+005
L	0	0	1e-6	6.69174e+005
L	0	0	1e-7	6.69174e+005
L	1e-5	1	1e-5	6.69174e+005 +3.34587e+000i

IV. CONCLUSIONS

In this paper, the first natural frequency of the lateral vibration modes of a test chip containing a micro-electro-mechanical system force sensor beam was analyzed. The major advantages of this work can be summarized as follows:

(1) Different materials result in different natural frequencies; the higher the Young modulus, the higher the natural frequency, which conforms to the vibration mechanics theory.

(2) When the spring and the damping are placed at the endpoints of the force sensor beam, there is no

change in the hysteretic damping ratio, and the spring constant and damping constant are sufficient enough to alter the first natural frequency of the system.

(3) When the spring and the damping are placed where $x_s = 0.5L, 0.75L$ m, the first natural frequency is in proportion to the spring constant, but in inverse proportion to the damping constant.

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BIOGRAPHIES

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