Time-Frequency Multiplex Estimator Design of Joint Tx IQ Imbalance, CFO, Channel Estimation, and Compensation for OFDM Systems

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***Abstract* —A low complexity time-frequency multiplex estimator and low complexity equalizer transceiver design is proposed to combat the RF impairment problems for the zero-IF transceiver of orthogonal frequency-division multiplexing (OFDM) systems. Moreover, the proposed preamble can estimate and compensate the transmitter (Tx) in-phase and quadrature-phase (IQ) imbalance, carrier frequency offset (CFO) and channel impulse response parameters. There are two parts in the proposed system. Firstly, all parameters of impairments are estimated by the proposed time-frequency multiplex estimator design. Secondly, the estimated parameters are used to equalize the above problems and detect the transmitted signal with low complexity advantage. Simulation results confirm that the proposed estimator can provide reliable performance over the severe IQ imbalance, CFO, and multipath fading channel environment[1].**

I. INTRODUCTION

Recently, due to the requirement of low power consumption, i.e., single chip design and low computational complexity, the zero-IF receiver has drawn a lot of attentions and exhibited great advantages in the high data rate wireless communication systems. As one of the most popular technologies, orthogonal frequency division multiplexing (OFDM) is widely adopted as the transmission scheme of choice for almost all broadband wireless standards, i.e., IEEE 802.11a/g, WiMAX, LTE, and Digital Video Broadcasting (DVB). However, the two types mainly of radio-frequency front-end imperfections are carrier frequency offset (CFO) and In-phase and Quadrature-phase (IQ) imbalance for OFDM communication systems [1]-[3]. Gain and phase mismatches in zero-IF transceiver can seriously degrade the system performance. CFO causes phase error and inter-carrier interference (ICI), and Tx IQ imbalance results in mirror frequency interference. In other words, CFO is induced by the transmitter (Tx) and receiver (Rx) with carrier frequency mismatch. Tx IQ imbalance is the up-conversion with the amplitude and phase mismatches between I- and Q-branch. Besides, multipath fading channel effects still cause the performance degradation of the OFDM transceiver. In summary, the performances of OFDM system are limited by the presence of multipath fading channel with Tx IQ imbalance and CFO effects. In order to remedy these problems, several schemes have been proposed to estimate and compensate for the RF impairment and channel effect, for example, [4]-[7]. The previous researches are based on the estimation and compensation for individual Tx IQ imbalance and CFO by using specific preamble [7]. However, in presence of Tx IQ imbalance, CFO, and channel, the methods are not optimal. To the best of my research, there is no publication paper proposed joint Tx IQ imbalance, CFO and channel estimators for zero-IF transceiver of OFDM systems.

The paper is organized as follows: In Section II, we formulate the transceiver signal model and propose the time-frequency multiplex preamble. In Section III, the estimator structure and the associated algorithms are developed by the proposed preamble. In Section IV, we introduce the proposed low complexity equalizer. Simulation results of the proposed systems are then demonstrated in Section V. In Section VI, some conclusions are made.

II. DATA MODEL

A. Impairment Model

Firstly, the OFDM transmitted signal in frequency domain is denoted by $\mathbf{x}$. The original signal is affected by the joint Tx IQ imbalance, CFO and multipath fading channels. Then, the received complex signal which removes the CP and transforms by FFT is given as [1]

$$\mathbf{y} = \alpha_1 \mathbf{GHx} + \alpha_2^* \mathbf{GHx}^\# \tag{1}$$

where $\alpha_1 = \left(1 + g_T e^{j\phi_T}\right)/2 \approx 1$ and $\alpha_2 = \left(1 - g_T e^{-j\phi_T}\right)/2 \approx 0$ with g_T and ϕ_T being gain and phase mismatch. $\mathbf{H}$ is a diagonal frequency response matrix. $\mathbf{G} = \begin{bmatrix} \mathbf{G}_1\mathbf{G}_2 \\ \mathbf{G}_2\mathbf{G}_1 \end{bmatrix}$ is a CFO matrix in frequency domain. The CFO circular matrix has the property of inverse operation, i.e., $\mathbf{G}^{-1} = \mathbf{G}^* = \mathbf{G}^H = \mathbf{FE}^*\mathbf{F}^H$ with $\mathbf{F}$ being FFT matrix and $\mathbf{E} = diag\left\{1, e^{j2\pi\varepsilon_f/N}, \ldots, e^{j2\pi(N-1)\varepsilon_f/N}\right\}$ denoted by the time-domain CFO matrix with ε_f being the normalized CFO. # operation represents conjugate mirror operation, i.e., the carriers take conjugate and turn upside down as follows.

$$\begin{cases} x^\#[k] = x^*[N-k] & \text{for } k = 1, \cdots, N-1 \\ x^\#[k] = x^*[k] & \text{for } k = 0 \end{cases} \tag{2}$$

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The relationship of α_1 and α_2 is given by

$$\alpha_1 + \alpha_2^* = 1 \tag{3}$$

$$\alpha_1 - \alpha_2^* = g_T e^{j\phi_T} = \mu_T \tag{4}$$

B. Preamble Design

Time-frequency-multiplex preamble design is a core key in the proposed transceiver of this paper. The preamble assists receiver to estimate parameters and overcome to image interference. That is, as shown in Fig. 1, the two contiguous preambles periodically in preamble 1 and 2 can be used to estimate CFO parameter via cross-correlation scheme. Next, IQ imbalance and channel parameters are estimated by two preambles with time-frequency-multiplex nulling techniques, i.e., negative subcarriers nulling in preamble 1 and positive subcarriers nulling in preamble 3. The nulling design can avoid the IQ imbalance interference. And the preamble sequence is designed by Chu-sequence, which contains the low PAPR and orthogonal property.

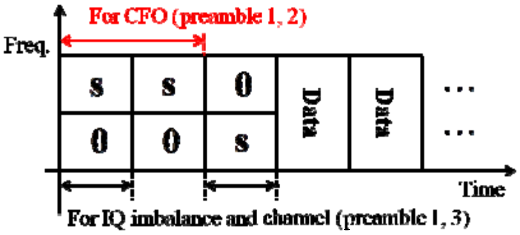


Fig. 1 Time-Frequency-Multiplex Preamble format.

The preamble $\mathbf{s}$ for suppressing the IQ imbalance is composed of two parts, the Chu-sequence and the conjugate Chu-sequence, i.e.,

$$\mathbf{s} = \begin{cases} \exp\left(jr\pi k^2/N\right) & \text{for k} \in \{1,\dots,N/4\} \\ \exp\left(jr\pi k^2/N\right)^* & \text{for k} \in \{N/4+1,\dots,N/2\} \end{cases} \tag{5}$$

The property of preamble is given by

$$\mathbf{s} = \mathbf{s}^\# \tag{6}$$

$$\mathbf{s}^* \circ \mathbf{s} = \mathbf{1} \tag{7}$$

where $\circ$ operation represents the element wise multiplication of vectors. And vector $\mathbf{1}$ is $N\times 1$ vector of all ones. Eq. (6) shows that the preamble with conjugate mirror operation is the same sequence. From Eq. (7), the estimator is easy to cancel the preamble by conjugate vector. In following sections, the time-frequency multiplex preamble estimates all impairment parameters.

III. DEVELOPMENT OF JOINT TX IQ IMBALANCE, CFO, AND CHANNEL ESTIMATION

A. Joint Tx IQ Imbalance, CFO, and Channel Estimator

Firstly, CFO effect is linear in time domain, so it can easily be estimated from the received preamble 1 and 2 that can be derived as

$$\mathbf{y}_1 = \begin{bmatrix} \mathbf{y}_{+,1} \\ \mathbf{y}_{-,1} \end{bmatrix} = \begin{bmatrix} \alpha_1 \mathbf{G}_1 \mathbf{H}_+ \mathbf{s} + \alpha_2^* \mathbf{G}_2 \mathbf{H}_- \mathbf{s} \\ \alpha_1 \mathbf{G}_2 \mathbf{H}_+ \mathbf{s} + \alpha_2^* \mathbf{G}_1 \mathbf{H}_- \mathbf{s} \end{bmatrix} \tag{8}$$

$$\mathbf{y}_2 = \begin{bmatrix} \mathbf{y}_{+,2} \\ \mathbf{y}_{-,2} \end{bmatrix} = \begin{bmatrix} \alpha_1 \mathbf{G}_1 \mathbf{H}_+ \mathbf{s} e^{j2\pi\varepsilon_f} + \alpha_2^* \mathbf{G}_2 \mathbf{H}_- \mathbf{s} e^{j2\pi\varepsilon_f} \\ \alpha_1 \mathbf{G}_2 \mathbf{H}_+ \mathbf{s} e^{j2\pi\varepsilon_f} + \alpha_2^* \mathbf{G}_1 \mathbf{H}_- \mathbf{s} e^{j2\pi\varepsilon_f} \end{bmatrix} \tag{9}$$

where the $+$ and $-$ represent the positive and negative subcarriers part in a vector. From Eq. (6), the conjugate mirror preamble $\mathbf{s}^\#$ can be replaced by $\mathbf{s}$. A complex scale is different between the two received preambles. The normalized CFO is estimated by the cross-correlation scheme. Then, the average of the positive and negative correlation results will be accurate for the CFO parameter estimation, i.e.,

$$\hat{\varepsilon}_f = \left(\measuredangle \mathbf{y}_{+,1}^H \mathbf{y}_{+,2} + \measuredangle \mathbf{y}_{-,1}^H \mathbf{y}_{-,2}\right) / 4\pi \tag{10}$$

Next, using the estimated normalized CFO can get the estimated CFO matrix as

$$\hat{\mathbf{G}} = \mathbf{F} diag\left\{\exp\left(j2\pi\hat{\varepsilon}_f n/N\right)\right\} \mathbf{F}^H \tag{11}$$

Secondly, using the estimated CFO matrix, the two unknown parameters, i.e., IQ imbalance and channel response, can be jointly estimated. That is, compensating CFO for the received preamble 1 signal cancels out the CFO impairment as

$$\begin{aligned} \mathbf{r}_1 &= \hat{\mathbf{G}}^* \mathbf{y}_1 \\ &= \alpha_1 \hat{\mathbf{G}}^* \mathbf{GHx} + \alpha_2^* \hat{\mathbf{G}}^* \mathbf{GHx}^\# \\ &= \alpha_1 \Delta\bar{\mathbf{G}} \mathbf{Hx} + \alpha_2^* \Delta\bar{\mathbf{G}} \mathbf{Hx}^\# \\ &= \begin{bmatrix} \mathbf{r}_{+,1} \\ \mathbf{r}_{-,1} \end{bmatrix} \cong \begin{bmatrix} \alpha_1 \mathbf{H}_+ \mathbf{s} \\ \alpha_2^* \mathbf{H}_- \mathbf{s} \end{bmatrix} \end{aligned} \tag{12}$$

where $\Delta\bar{\mathbf{G}} = \hat{\mathbf{G}}^* \mathbf{G} \cong \mathbf{I}_{N\times N}$ with the accurate CFO estimation assumption. By the same way, the received preamble 3 vector is compensated by the CFO matrix as

$$\mathbf{r}_3 = \hat{\mathbf{G}}^* e^{-j4\pi\varepsilon_f} \mathbf{y}_3 = \begin{bmatrix} \mathbf{r}_{+,3} \\ \mathbf{r}_{-,3} \end{bmatrix} \cong \begin{bmatrix} \alpha_2^* \mathbf{H}_+ \mathbf{s} \\ \alpha_1 \mathbf{H}_- \mathbf{s} \end{bmatrix} \tag{13}$$

Next, using the properties of preamble design in Eq. (7) and the IQ imbalance parameters in Eq. (3), the positive and negative frequency-domain channel vectors are estimated by

$$\hat{\mathbf{\Lambda}}_+ = \left(\tilde{\mathbf{r}}_{+,1} + \tilde{\mathbf{r}}_{+,3}\right) \circ \mathbf{s}^* \tag{14}$$

$$\hat{\mathbf{\Lambda}}_- = \left(\tilde{\mathbf{r}}_{-,1} + \tilde{\mathbf{r}}_{-,3}\right) \circ \mathbf{s}^* \tag{15}$$

From Eqs. (14) and (15), the frequency-domain channel matrix is easily reconstructed as $\hat{\mathbf{H}} = diag\{\begin{bmatrix} \hat{\mathbf{\Lambda}}_+ & \hat{\mathbf{\Lambda}}_- \end{bmatrix}^T\}$.

According to Eqs. (12) and (13), Tx IQ imbalance parameters can be obtained from the cross-correlation. Then, the average processing of channel estimation of the positive and negative subcarrier parts will increase the estimated accuracy of the IQ imbalance parameters. It is derived as

$$\hat{\alpha}_1 = \left(\hat{\mathbf{\Lambda}}_+^H \left(\mathbf{r}_{+,1} \circ \mathbf{s}^*\right) + \hat{\mathbf{\Lambda}}_-^H \left(\mathbf{r}_{-,3} \circ \mathbf{s}^*\right)\right) \Big/ \left(\left\|\hat{\mathbf{\Lambda}}_+\right\|^2 + \left\|\hat{\mathbf{\Lambda}}_-\right\|^2\right) \tag{16}$$

$$\hat{\alpha}_2 = \left(1 - \hat{\alpha}_1\right)^* \tag{17}$$

B. Iterative Tx IQ Imbalance, CFO, and Channel Fading Estimators

To enhance the accuracy of the estimated parameters, the iteration processing can be used. First, the channel and Tx IQ imbalance parameters estimation can be iterated by cancelling the image interference, i.e., the received signal with the interference cancellation and CFO compensation being expressed by

$$\begin{aligned}\mathbf{r}^{(i+1)} &= \hat{\mathbf{G}}^{(i)*}e^{-j2\pi(l-1)\hat{\varepsilon}_f^{(i)}}\left(\mathbf{y}-\hat{\alpha}_2^{(i)*}\hat{\mathbf{G}}^{(i)}\hat{\mathbf{H}}^{(i)}\mathbf{x}^{\#}\right) \\ &\cong \alpha_1\mathbf{H}\mathbf{x}\end{aligned} \tag{18}$$

where (i) is the number of iteration, i.e., $i=0$ is without iteration. Next, considering the assumption of the accuracy parameters estimation, the iterative preamble 1 and 3 can be given as

$$\mathbf{r}_1^{(i+1)}=\begin{bmatrix}\mathbf{r}_{+,1}^{(i+1)}\\ \mathbf{r}_{-,1}^{(i+1)}\end{bmatrix}\cong\begin{bmatrix}\alpha_1\mathbf{H}_+\mathbf{s}\\ 0\end{bmatrix},\ \mathbf{r}_3^{(i+1)}=\begin{bmatrix}\mathbf{r}_{+,3}^{(i+1)}\\ \mathbf{r}_{-,3}^{(i+1)}\end{bmatrix}\cong\begin{bmatrix}0\\ \alpha_1\mathbf{H}_-\mathbf{s}\end{bmatrix} \tag{19}$$

The parameters of Tx IQ imbalance and channel can be estimated with more accuracy from the Eq. (19), which is the same as the procedures in Eqs. (14)-(17).

IV. LOW COMPLEXITY EQUALIZER DESIGN WITH TX IQ IMBALANCE, CFO, AND FADING CHANNEL EFFECTS

In a standard OFDM receiver without RF distortion, single tap equalizers are sufficient for channel equalization, but it is not capable of removing the distortions, i.e., IQ imbalance and CFO. It is due to that the conventional OFDM receivers do not take into account the image subcarriers which are generating the additive interference component. In presence of Tx IQ imbalance and CFO, the OFDM equalizer needs more complex equalization schemes. First, the received signal is compensated by the estimated CFO in frequency domain as follow

$$\mathbf{r}=\hat{\mathbf{G}}^{*}e^{-j2\pi(l-1)\hat{\varepsilon}_f}\mathbf{y}=\begin{bmatrix}\mathbf{r}_+\\ \mathbf{r}_-\end{bmatrix}\cong\begin{bmatrix}\alpha_1\mathbf{H}_+\mathbf{x}_++\alpha_2^*\mathbf{H}_+\mathbf{x}_-^{\#}\\ \alpha_1\mathbf{H}_-\mathbf{x}_-+\alpha_2^*\mathbf{H}_-\mathbf{x}_+^{\#}\end{bmatrix} \tag{20}$$

Next, from the compensated CFO signal, the negative part uses conjugate mirror operation to get the decomposition of the matrix into impairment matrix and transmitter signal vector, i.e.,

$$\begin{bmatrix}\mathbf{r}_+\\ \mathbf{r}_-^{\#}\end{bmatrix}=\begin{bmatrix}\alpha_1\mathbf{H}_+ & \alpha_2^*\mathbf{H}_+\\ \alpha_2\mathbf{H}_-^{\#} & \alpha_1^*\mathbf{H}_-^{\#}\end{bmatrix}\begin{bmatrix}\mathbf{x}_+\\ \mathbf{x}_-^{\#}\end{bmatrix} \tag{21}$$

The size of the composite channel matrix in Eq. (21) is $N\times N$. The complexity of maximum likelihood (ML) equalizer is very high. However, the matrix can be divided into $N/2-1$ groups of 2×2. The complexity of 2×2 inverse matrix is very low and easy to obtain. The effects on the corresponding pairs of transmitted symbols can be expressed by a matrix $\mathbf{A}$ as follows

$$\underbrace{\begin{bmatrix}\mathbf{r}[v]\\ \mathbf{r}^*[u]\end{bmatrix}}_{\tilde{\mathbf{r}}[v]}=\underbrace{\begin{bmatrix}\alpha_1\mathbf{H}_1[v] & \alpha_2^*\mathbf{H}_1[v]\\ \alpha_2\mathbf{H}_2^{\#}[u] & \alpha_1^*\mathbf{H}_2^{\#}[u]\end{bmatrix}}_{\mathbf{A}[v]}\underbrace{\begin{bmatrix}\mathbf{x}[v]\\ \mathbf{x}^*[u]\end{bmatrix}}_{\tilde{\mathbf{x}}[v]} \tag{22}$$

where $v\in\{1,\cdots N/2-1\}$, $u=N-v$. Based on the equalizer model, $\tilde{\mathbf{x}}[v]$ can be obtained by the common equalization methods, such as zero forcing (ZF) and ML. And the transmitted data can be reconstructed by $\tilde{\mathbf{x}}[v]$.

V. COMPUTER SIMULATIONS

In the section, simulation results are conducted to demonstrate the performance of the proposed time-frequency multiplex estimator and equalizer. Monte Carlo simulations are performed. The parameters used in the simulation are the length of OFDM symbol being N = 64 and the length of cyclic prefix being N/4. The time-frequency multiplex preamble design proposed in Section II is used for Tx IQ imbalance, CFO, and channel estimation. Channel profile is an equal gain multipath channel with 4 taps where the taps are chosen independently with complex Gaussian distribution which is variable with a zero-mean and unit variance. Also, the normalized frequency offset is assumed to be $\varepsilon_f=0.2$. The amplitude and phase mismatch of Tx IQ imbalance are chosen to be $g_T=1.1$ and $\phi_T=5°$ or $10°$. Note that all Tx IQ imbalance and CFO parameters are assumed to be time-invariant. As a performance index, the simulation results of bit error rate (BER) and mean square error (MSE) versus signal-to-noise ratio (SNR) are evaluated to confirm the performance of the proposed system. The MSE is defined as

$$\mathrm{MSE}_h=\frac{1}{MN}\sum_{i=1}^{M}\left\|\hat{\mathbf{\Lambda}}^{(i)}-\mathbf{\Lambda}\right\|^2 \tag{23}$$

where M is the total of Monte-Carlo trials.

In the first set of simulations, Fig. 2 shows that the MSE of channel compares between different Tx IQ imbalance effects as function of E_b/N_0. Three MSE curves show the accuracy of the estimation. The estimated results are robust under the different serious IQ imbalance cases. As shown in Fig. 2, the performance of channel estimation improves considerably as iteration increases, and the twice iteration is enough.

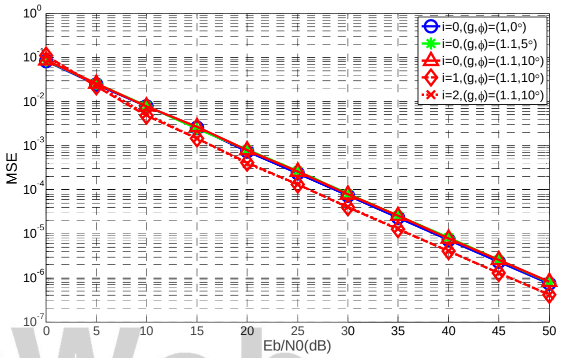
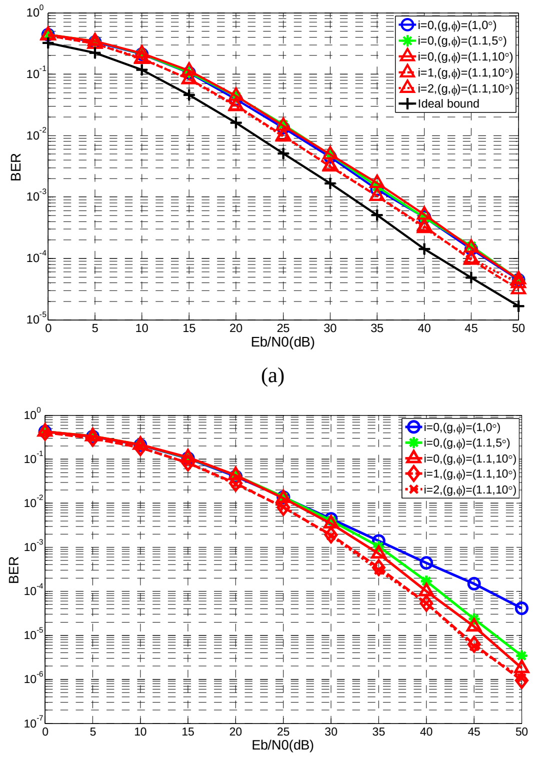


Fig. 2 MSE of channel versus Eb/N0 for proposed iterative estimators with different IQ imbalance and iterative times under $\varepsilon_f=0.2$.

In the second set of simulations, Fig. 3 shows the BER curves of the proposed low complexity ZF and ML equalizers for the same environment as the first simulation.

We use 16-QAM modulation, which is sensitive to non-ideal effects. The ZF equalizer is robust in different IQ imbalance situations. However, as shown in Fig. 3, the BER performance in ML equalizers is better under the more severity of IQ imbalance scenario. It is due to that the Tx IQ imbalance in Eq. (22) causes the equivalent model of $N/2-1$ pairs of subcarriers which can obtain the additional frequency diversity [6]-[7] under the serious IQ imbalance effect. It can be more clearly observed in the next simulation. In all figures, “Ideal bound” legend refers to a receiver with perfect IQ imbalance, CFO and channel knowledge.



(a)

(b)

Fig. 3 BER versus Eb/N0 for proposed (a) ZF and (b) ML equalizers with different IQ imbalance and $\varepsilon_f = 0.2$.

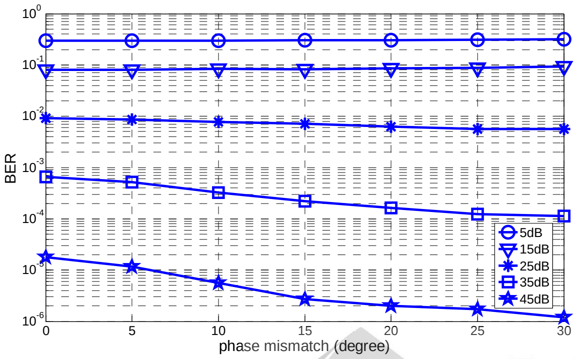


Fig. 4 BER versus phase mismatch for proposed receiver with different Eb/N0 under $g_T = 1.1$ and $\varepsilon_f = 0.2$.

In the third set of simulations, simulation investigates the robust BER performance of the proposed ML equalizer as a function of phase mismatch and normalized CFO effects. The results are shown in Fig. 4 and Fig. 5. Setting the simulation parameters, the comparisons change one of parameters of the Tx IQ imbalance and normalized CFO. As observed, the proposed receiver can provide the reliable BER performance under different cases. Fig. 4 shows that the performance improves under the Tx IQ imbalance effect increased. From Fig. 5, we can find that the BER results with the reliable performance only guarantee to estimate in range of ±0.5 for the proposed estimator. The phase ambiguity problems are observed in the Eqs. (8) and (9). The phase useful range is $-\pi < 2\pi\varepsilon_f < \pi$. On the other hand, the normalized CFO range is within ± 0.5 .

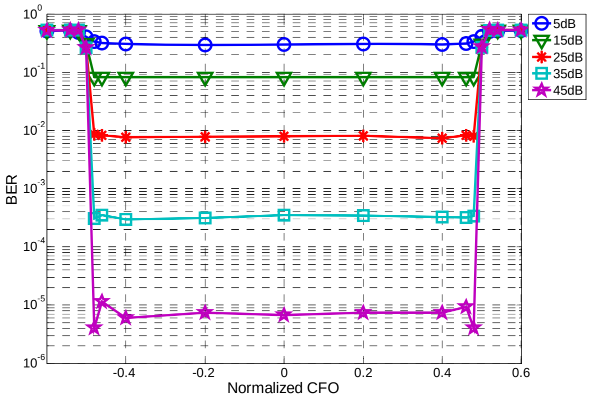


Fig. 5 BER versus normalized CFO for proposed receiver with different E_b/N_0 under $g_T = 1.1$ and $\phi_T = 10°$.

VI. CONCLUSION

This paper studied the problem of Tx IQ imbalance and CFO under the multipath channel in OFDM systems. Based on the proposed time-frequency multiplex preamble, the transceiver has been developed to estimate and compensate for such distortions. Finally, simulation results show that the proposed low complexity estimator and ML equalizer with frequency diversity gain have robust performance.

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