The Application of Receiver Operating Characteristics to Improve Mobile Subscriber Equipment

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Abstract
Improved Mobile Subscriber Equipment (IMSE) provides probable detection of, and access to, a mobile communication system in emergency situations. Sometimes one piece of IMSE can receive multiple signals at the same time. In such situations, the IMSE needs to identify the required signal. In this study, we use the receiver operating characteristic (ROC) curve to indentify the required signal. In most cases, a receiver can detect a strong and clear signal which has a high signal to noise ratio and receiving samples. For example, when the signal to noise ratio (SNR) increases from 0.23 to 0.75, it shows that the probability of detection also increases from 0.8433 to 0.9999 in the same sample. On the other hand, in the same SNR situation of 0.75, for example, when receivers gain signal samples from 20 to 200, the probability of detection increases at the same time. Thus, receivers are able to easily identify the signal with the highest number of samples and SNR if there are two or more signals being received at the same time, and then use ROC curves to show their probability of detection and probability of false alarm for the target signal.

I. INTRODUCTION

Currently, improved mobile subscriber equipment (IMSE) is being extensively used in military and emergency rescue. As an example, after earthquakes in Japan, many communication systems may be destroyed by tidal waves. In such situations, IMSE can be used to build a temporary communication network, thus making it possible to gather information on how bad the situation is in various areas affected by the disaster. It is therefore essential that IMSE is able to accurately detect a signal and send precise information to the command center to help in decision making.

However, we first need to determine if IMSE has good signal detection performance under certain conditions. In this project, we apply ROC to study the signal detection performance of IMSE. First, we briefly describe the IMSE operation, and then introduce the relation of detection problems between IMSE and ROC. Finally, numerical examples are simulated by Matlab software as a discussion case [5].

II. THE OPERATION OF IMSE

Generally, IMSE consists of two main components: a switchboard system and a multi-frequency channel system [1]. The switchboard system consists of a large screen showing the connections between each IMSE unit. The multi-frequency channel system is responsible for signal transmission and reception via a module-128(CTM-128) communication terminal, which is a radio-wave frequency switcher. By means of CTM-128, we can choose a specific desired frequency, such as ultra high frequency (UHF) or very high frequency (VHF) [2].

Once there is a signal from one of the nodes, an alarm signal will be represented on the monitor. From this signal, it is possible to learn which nodes want to send messages to the main node. The main node may transmit a reply signal to one of those nodes. When the IMSE receives the reply signal, it starts transmitting messages to the main node. In order to determine exactly which node is sending messages, the main node signal detector must have a high signal detection probability, which means that detecting false alarms must be constrained under a certain value. For a given probability of false alarms, the signal detection performance of the detector is evaluated by its corresponding ROC curves.

III. CHARACTERISTICS OF THE ROC CURVE

ROC curves plot the relation probability between the detection and false alarms of a signal detector; the signal detection performance of the detector is thereby calculated. According to the curve, we can maximize the detection probability of the detector by methods such as the Neyman-Pearson (NP) theorem. Figure 1 shows a basic ROC curve [1][5].

Each point on the curve represents a pair of detection and false alarm probabilities for a given threshold γ from a signal detector. In our case, we set $\mu = \frac{SNR}{N}$ as a threshold in γ. From figure 2, if we set probability of detection as a constant, we can know that when γ increases, the probability of false alarm (Pf) decreases. This is the standard type of receiver operating characteristic curve. As shown in Fig. 1, the gamma depends on the false alarm and detection probability values. To set any threshold γ, two kinds of probabilities are determined.

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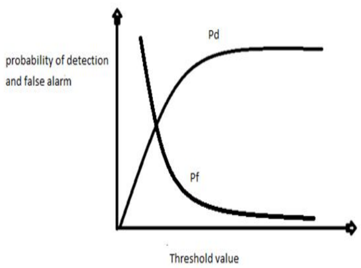

Fig. 1: The standard Receiver Operating Characteristic curve. The X axis is the threshold value, the Y axis is the probability of detection and false alarm, Pd is probability of detection and Pf is probability of false alarm.

In the curve, if we define probability of detection as the probability that the main node will identify signals as coming from node 2, for example, this means that the main node accurately receives signals from node 2. The probability of false alarm is the probability that the main node will indentify a signal from node 1 as coming from node 2, which assumes that the main node has already received a signal from node 2, although it did not.

In practice, signal transmission efficiency depends on the position of the IMSE. If node 2, for instance, is in a mountain valley while node 1 is set up on a plain, when they transmit signals to the main node at the same time, it is clear that the signal from node 1 will have a higher detection probability than that of node 2, because the signal from node 2 will have a lower signal strength than that of node 1. The following is an example of detection and false alarm probabilities being determined.

First, the Neyman-Pearson theory is derived Then, using its result, the threshold value of the detector and the maximum likelihood ratio function are obtained [3]-[5].

Lagrangian multipliers are presented to obtain the maximal detection probability value (Pd) for a given probability of false alarm (Pf). The Lagrangian formula is defined as: $F = Pd + \lambda(Pf - \alpha)$, where λ is a Lagrange multiplier, and α is a constant. Then,

$$F = Pd + \lambda(Pf - \alpha) = \int_{R1} p(x;H_1)dx + \lambda(\int_{R1} p(x;H_0)dx - \alpha) =$$

$$= \int_{R1} (p(x;H_1) + \lambda p(x;H_0))dx - \lambda\alpha \tag{1}$$

$p(x;H_1)$ is the probability density function (pdf) of x when H_1 is true.

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To maximize F, it should include x in Region 1 where H_1 is in the Region 1 (R1), if the integrand is positive for that value of x, or if,

$$p(x;H_1) + \lambda p(x;H_0) > 0 \tag{2}$$

Then we obtain: H_1 if $\dfrac{p(x;H_1)}{p(x;H_0)} > -\lambda$.

Finally, let $\gamma = -\lambda$ so the likelihood ratio function is H_1 if $\dfrac{p(x;H_1)}{p(x;H_0)} > \gamma$ when $pf = \alpha$.

The above theory shows that the signal is a direct current in White Gaussian Noise from node 1, represented as $w_1[n]$:

$$H_0 : x_0[n] = w_1[n], n = 1,2......N \tag{3}$$

The signal from node 2 is the desired signal, represented as $A[n]$,

$$H_1 : x_1[n] = A[n] + w_2[n], n = 1,2,3........N \tag{4}$$

where $w_n[n]$ is the noise from node 2, and α is constant.

In the above equation, $A[n]$ is the desired signal, and $w_2[n]$ is the noise from node 2. Their Gaussian pdfs can be shown as:

$$p(x_0[n];H_0) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-x_0[n]^2}{2\sigma^2}} \tag{5}$$

$$p(x_1[n];H_1) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x_1[n]-A[n])^2}{2\sigma^2}} \tag{6}$$

According to the NP theorem, the likelihood ratio functions of signals from nodes 1 and 2 are:

$$L(x) = \frac{P(x_1[n];H_1)}{P(x_0[n];H_0)} = e^{\frac{-(x_1[n]-A[n])^2}{2\sigma^2} \geq \gamma}, underH_1$$ or

$$L(x) < \gamma, underH_0$$

where γ is the threshold value of the detector. Then, Pd (probability of detection) and Pf (probability of false alarm) for an ideal case are described as:

$$Pf = P(H_1;H_0) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-x_0[n]^2}{2\sigma^2}} d(x_0[n]) = \alpha \qquad (7)$$

and

$$Pd = P(H_1;H_1) = \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x_1[n]-A[n])^2}{2\sigma^2}} d(x_1[n]) = 1-\alpha$$

Therefore, the Pf and Pd of the detector can be calculated using the above definition; the corresponding ROC curves can be plotted [5].

IV. NUMERICAL SIMULATION

If the SNR (signal to noise ratio) of the case is defined as:

$$\frac{1}{\sigma^2}\sum_{n=1}^{N} |A[n]|^2 \qquad (8)$$

and the mean value of the SNR is $\mu = \frac{SNR}{N} = 0.75$ and 0.23 respectively, with two samples of N = 20 and N = 200, the ideal ROC (approaching the ideal ROC curve) and normal ROC curves of this case are as shown in Figure 2, in which N = 30, 40, 60 and 200 are plotted in dash lines, and N = 20 is the star line.

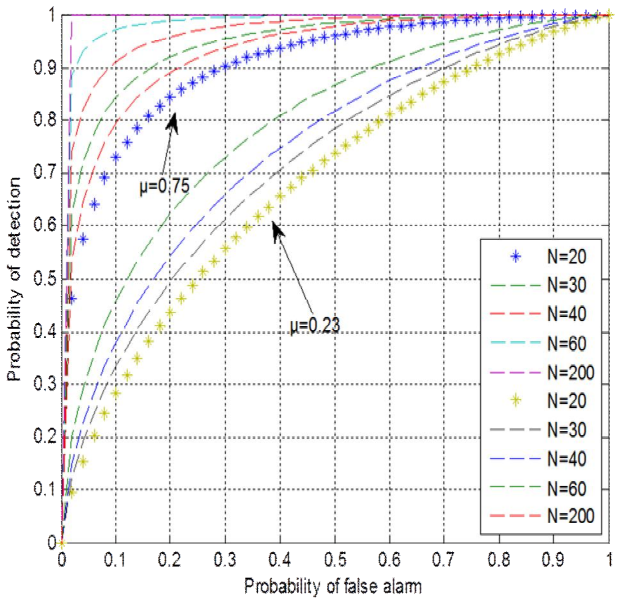

Fig. 2: Probability of detection vs. probability of false alarm with different N and μ sample values

As shown in Figure 2, the probability of detection of N = 200 is very close to 1 when the probability of false alarm approaches 0 and the value of μ approaches 1. However, under certain constrained probability of false alarm values, different samples relate to different values of probability of detection. The more samples taken, the higher the detection probability attained will be.

Therefore, an alternative solution for the previous example shows that $Pf = Q(\frac{\gamma}{\sqrt{\sigma^2/N}})$ and $Pd = Q(\frac{\gamma - A[n]}{\sqrt{\sigma^2/N}})$.

So the relation between Pf and Pd is: $Q(Q^{-1}(Pf) - \sqrt{d^2})$, where $d^2 = \frac{N(A[n]^2)}{\sigma^2}$. Q^{-1} is the complementary cumulative distribution function, γ the threshold of each ROC curve, $A[n]$ is the signal without noise and n is the number of samples that nodes receive. When d increases, Pf decreases and Pd increases. This means that the threshold value of γ also increases. This results in more ideal ROC curves and higher probability detection of incoming signals. In conclusion, analyzing the characteristic of a signal's ROC curve is a convenient way to study the signal detection performance of a detector, as shown in Fig. 3, when μ = 0.75.

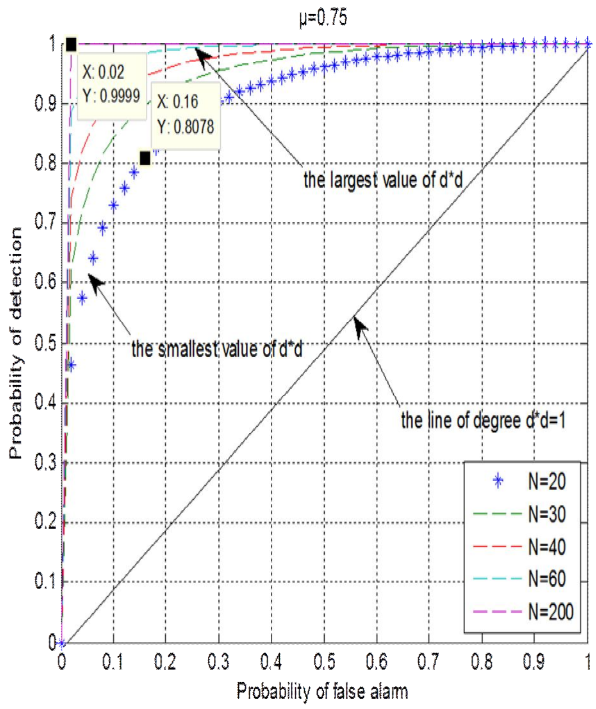

Fig. 3: Different threshold values for different ROC curves.

I. CONCLUSION

From the above discussion, IMSE essentially operates as a bilateral wireless communication system. However, when two (or more) units, such as node 1 and node 2, simultaneously send messages to the main node, a hypothesis testing problem arises, requiring the main node to accurately determine the source node of each signal. The NP theorem may be used to set the threshold value of the main node signal detector with a certain false alarm probability, and obtain the detection probability. By plotting the corresponding ROC curve, the detection performance of the detector can be obtained.

From Table 1, when a receiver receives a greater number of samples and the signal to noise ratio is higher, the probability of detection increases and the probability of false alarm decreases. Therefore, the incoming signal will be clearly detected by receivers, and will not be misunderstood.

Table 1: Comparison of different Receiver Operating Curves of Improved Mobile Subscriber Equipment receiving different numbers of samples and signal to noise ratios

μ = 0.75

Samples	Probability of detection	Probability of false alarm
N=20	0.7859	0.14
N=30	0.8431	0.1
N=200	0.9999	0.02

μ = 0.23

Samples	Probability of detection	Probability of false alarm
N=20	0.6178	0.36
N=30	0.6336	0.32
N=200	0.8433	0.14

REFERENCES

[1] N. A. Touhami, M. Boussouis, A. Tazon and A. Tribak. High Performance Rx/Tx Antenna for Wireless Applications. In ICMCS International Conference on Multimedia Computing and Systems,2012.

[2] R. Ramirez-Gutierrez, L. Zhang, J. Elmirghani, and A. F Almutairi. Antenna Beam Pattern Modulation for MIMO Channels. In IWCMC eighth International Wireless Communications and Mobile Computing Conference ,2012, pages 591-595

[3] Ha Anh Tuan Nguyen, Sy Vinh Le ,and Si Quang Le. A Maximum Likelihood Method for Detecting Bad Samples from Illumina BeadChips Data. In fourth International Conference on Knowledge and Systems Engineering,2012 pages 26-33.

[4] T. Li, J. Yang, and Z. Chen. The Early Warning and Prediction Method of Flea Beetle Based on Maximum Likelihood Algorithm Ensembles. In sixth International Conference on Natural Computation,ICNC/2010,pages 1901-1905.

[5] Steven M. Kay, Fundamentals of Statistical Signal Processing, Volume-II . Upper Saddle River, NJ: Prentice-Hall.