

# A Simple Method to Estimate the Propagation Constant of Microstrip line and its Application on the Design of Microstrip Leaky Wave Antennas

Jeng-Hau Lu\*<sup>a</sup> and Christina F. Jou<sup>a</sup>

*Institute of Communications Engineering, National Chiao Tung University<sup>a</sup>*

**Abstract** —A simple and efficient method to estimate the propagation constant of microstrip lines is proposed in this paper. By making use of the structural symmetry of microstrip lines, the insertion of PEC (PMC) symmetry walls can force the microstrip line to operate at a single 1<sup>st</sup> higher order (quasi TEM) mode. The corresponding complex propagation constant can be estimated from the S parameters using the multi-line method with the aid of full-wave simulators. The method can be a valuable tool for the accurate design of microstrip leaky wave antennas.

## I. INTRODUCTION

When using transmission line structures to guide the wave in the circuit, the circuit designer should always be aware of the possible excitation of higher order modes and the corresponding impact on the system. As pointed out by Oliner for the case of microstrip lines [1], the 1<sup>st</sup> higher order mode can excite "leaky wave" and the power is "leaked" toward free space from the aperture of the microstrip line. As a circuit designer, one may want to suppress such kind of "leakage loss" and prevent the cross talk in terms of the radiation coupling. On the other hand, one can also take advantage of the nature that the power can be leaked from the transmission line, which can be useful in antenna design. In any way, the characteristics of the modes that may be present in the transmission line structure should be carefully analyzed in order to know their impact on the functionality of circuit. This purpose of this paper is to demonstrate a method to estimate the propagation constant that the 1<sup>st</sup> higher mode of the microstrip line is a fast wave and the microstrip can contribute space wave leakage radiation [1]. This analysis can be useful in the design of microstrip leaky wave antennas [1-3].

Early approaches to calculate the propagation constant of a transmission line requires a full-wave formulation and root-searching algorithms to solve a 2-D eigenvalue problem [4, 5]. Although the method is rigorous, the implementation can be difficult for novice microwave circuit designers. On the other hand, measurement based method [6-10] can be very attractive since only simple post-processing of the S-parameters are needed to estimate the propagation constant. However, the transmission line should maintain single mode operation or the modal mutual couplings would complicate the post-processing procedures [7].

In this paper, the concept of numerical measurements [10] is introduced by simulating multiple transmission line structures as if the designer have them fabricated. The problem of the simultaneous presence of multiple modes have

been avoided by placing symmetry planes at the center of the microstrip line. The propagation constant can be extracted from the simulated S-parameters using the multi-line method.

The organization of this paper is as follows: A brief introduction of the multi-line method is given in section II. The application of the method on the analysis of microstrip line is introduced in section III. The results are summarized in section IV and a conclusion is made in section V.

## II. THE NUMERICAL MULTI-LINE METHOD

The numerical multi-line method begin with the procedure by considering two transmission lines with different longitudinal length  $l = L_{leakline} = l_i, l_j$  ( $l_j > l_i$ ) and the "measurement" of S-parameters by HFSS [11]. As depicted in Fig. 1, it is assumed in this paper only a single mode is present in the line, and the structure can be considered as a lossy transmission line cascade with error adapters that represents the transition between the 50  $\Omega$  port and the transmission line to be analyzed. This assumption should always be examined to make the following analysis valid. The following computations is programmed in MATLAB [12].

To facilitate the analysis, the S-parameter is first converted into the wave cascade matrix:

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} S_{12}S_{21} - S_{11}S_{22} & S_{11} \\ -S_{22} & 1 \end{bmatrix} \quad (1)$$

which the definition is follow the convention in [6]. The wave cascade matrices  $T_i$  and  $T_j$  which correspond to the two simulated transmission line structures can be expanded as:

$$T_i = T_A D_i T_B \quad (2)$$

$$T_j = T_A D_j T_B \quad (3)$$

where the matrices  $T_A$  and  $T_B$  represents the transition from port 1 to the beginning of the line and the end of the line to the port 2, respectively. The matrix  $D_i$  and  $D_j$  are diagonal matrices filled with different complex phase shift of the lines. The next step is the calculation of matrix  $M$  [6]:

$$M = T_j T_i^{-1} = T_A D_\Delta T_A^{-1} \quad (4)$$

Where  $D_\Delta = D_{line} D_{thru}^{-1}$  is a diagonal matrix filled with the complex phase difference of the two transmission line structures. To be explicit [8],

$$D_{\Delta,11} = e^{-\gamma(l_j - l_i)}, D_{\Delta,22} = e^{+\gamma(l_j - l_i)} \quad (5)$$

where  $\gamma$  is the propagation constant to be estimated in this paper. One can calculate the complex phase shift  $P_{ij}$  using

$$P_{ij} \triangleq \gamma(l_j - l_i) = [\ln(D_{\Delta,22}) - \ln(D_{\Delta,11})] / 2 \quad (6)$$

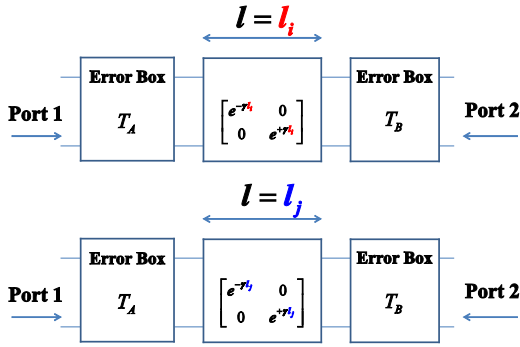


Fig. 1 The equivalent 2-port network of the transmission line

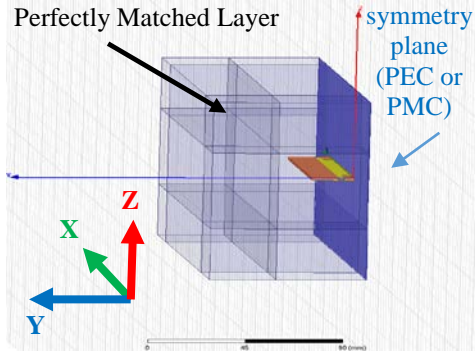


Fig. 2 The simulated structure of the microstrip line

where the average of the diagonal term is used instead for noise suppression. It is worth noting that calculated phase shift should be “unwrapped” to avoid the phase ambiguity problem when the phase exceed  $360^\circ$ , which can simply done by the MATLAB function `unwrap()` [12].

In order to find  $D_\Delta$ . One can rearrange the equation as [6]

$$MT_A = T_A D_\Delta \quad (7)$$

and one can then find the matrix  $T_A$  is the eigenvector of the matrix  $M$  with the eigenvalue which is exactly the diagonal terms in  $D_\Delta$ . The problem now reduces to the eigenvalue problem of matrix  $M$ , which can calculated with the aid of MATLAB function `eig()` [12]. Once the eigenvalue is solved, the complex phase shift can then be calculated from (6).

The above mentioned procedure is sufficient to estimate the propagation constant in TRL procedures [6] by dividing the previously solved complex phase shift by the difference of the length pair  $l_j - l_i$ . However, as pointed out by Marks [8], the calculated complex phase shift may be noisy that the error can be minimize when the phase shift is near  $\pm 90^\circ$ . If one can “measure” multiple sets of the transmission line with different length, one can make a better estimate by weighting the calculated results. In the multi-line method, an estimate of  $\gamma$  can be done by calculating [9]

$$\gamma_{est} = \frac{L^H U G}{L^H U L} \quad (8)$$

where the superscript  $H$  is the hermitian transpose, and

$$L = [\dots, \Delta l_{ij}, \dots]^T \quad (9)$$

$$G = [\dots, P_{ij}, \dots]^T \quad (10)$$

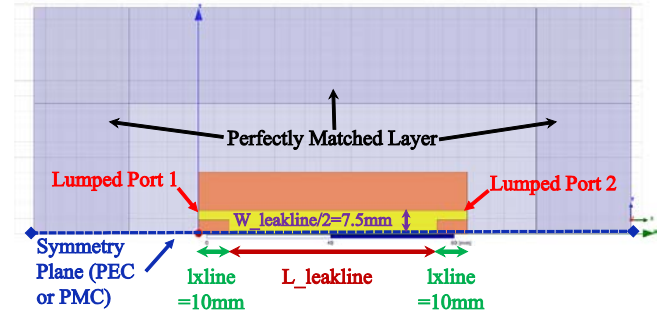


Fig. 3 The detailed structure overview

are the column vectors filled with the length difference  $l_j - l_i$  of the line pair  $i, j$  and corresponding phase difference, respectively. The matrix  $U$  with its matrix element

$$U_{mn} = \delta_{mn} - 1 / (N + 1) \quad (11)$$

( $\delta_{mn}$  is the kronecker delta) is a  $N \times N$  matrix used in [9] to weight and estimate  $\gamma_{est}$ .  $N$  is the number of line pairs considered in the estimation. The strategy to choose length of the line follows the suggestion in [8], which will discussed in detail in the next section.

### III. SIMULATION ISSUES IN THE MULTI-LINE METHOD

After a simple review on the multi-line method is done, the focus now turn to the considerations to setup the simulation for estimating the propagation constant of the microstrip line. The structure to simulate the transmission characteristic of microstrip line is shown in Fig. 2 and a detailed description is given in Fig 3. The full microstrip width  $w_{leakline}$  is 15 mm using FR4 substrate with substrate height  $h = 1.6$  mm and permittivity  $\epsilon_r = 4.4(1 - 0.02j)$ . The excitation of the microstrip line at the 2 ports is depicted in Fig. 3 by using a  $50 \Omega$  microstrip line.

As shown in Fig. 2, a symmetry PEC wall can be placed at the center of the microstrip to suppress the excitation of the fundamental quasi TEM mode and higher order even modes. If one is interest at the characteristics of the quasi TEM mode on the microstrip, one can simply change the boundary condition of the wall to a PMC to suppress all the odd modes. However, one should still notice that when the frequency is higher than the cut-off frequency of the second higher order odd (even) modes, the microstrip line cannot be guarantee to be operated at a single mode and the above analysis cannot be applied.

The proposed method can suit the need to analyze the microstrip line at the frequency range that the 1<sup>st</sup> higher order mode is a fast wave, which only the simultaneous presence of the quasi-TEM and the 1<sup>st</sup> higher order mode should be considered. In [3], a microstrip line with its width = 15 mm is analyzed and the frequency scanning range that having the previously mentioned characteristics is determined to be 4.2 GHz ~ 4.9 GHz. This frequency range will be the focus of the later analysis.

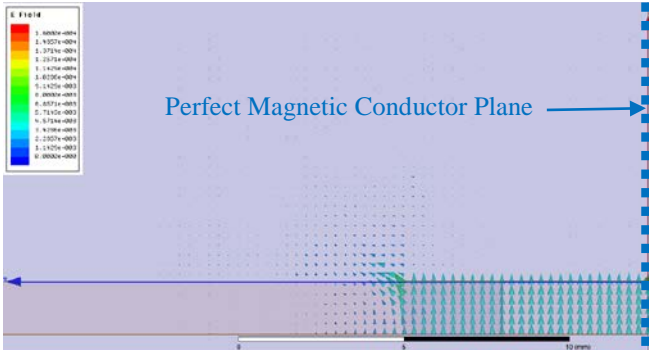


Fig. 4 The halved quasi TEM mode of the microstrip line

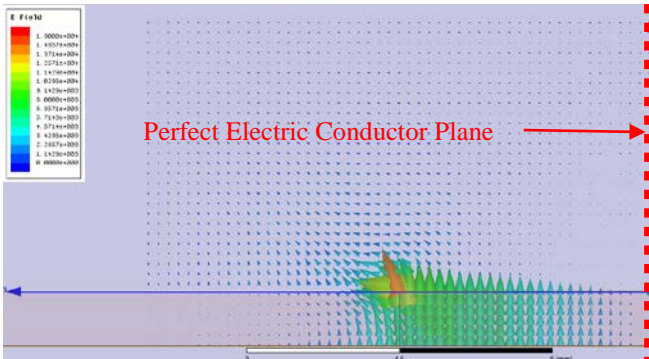


Fig. 5 The halved 1st higher order mode of the microstrip line

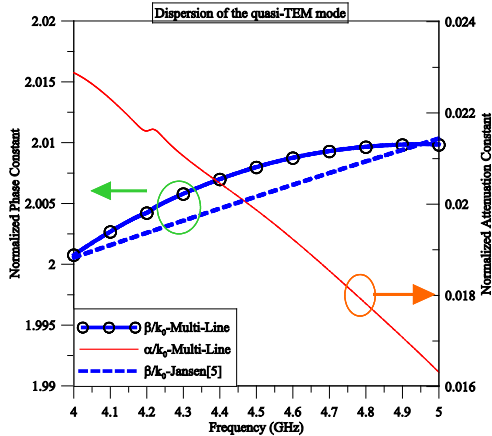


Fig. 6 The complex propagation constant of the quasi TEM mode

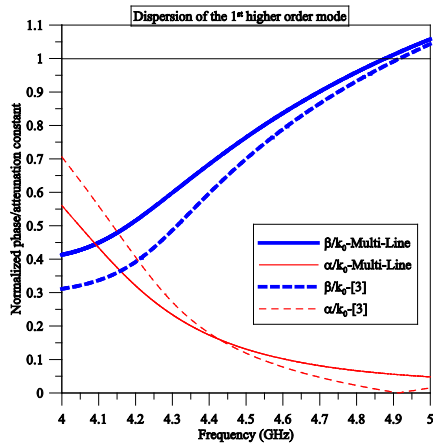


Fig. 7 The complex propagation constant of the 1st higher order mode

In this paper, three transmission line are used in the simulation for the purpose of demonstration, and there are  $C_2^3 = 3$  possible pairs to do the estimation. One can simulate more transmission line structures to make a better estimation. However, as suggested in [8], these  $C_2^N$  sets of line pairs are noisy and their error are linear dependent. So one should choose the portion of line pairs that their error are linear independent [8].

The line length  $l = 22, 37, 68$  mm which are denoted as  $l_1, l_2, l_3$  is used to calculate the propagation constant of the 1st higher order mode and only the line pair  $l_3 - l_1, l_2 - l_1$  is used in the estimation. This choice is made that the phase shift of  $l_1$  is near  $90^\circ$  at the band center 4.55 GHz which has been predicted previously in [3] and one of the line  $l_3$  is about 3 times of  $l_1$ . It is worth noting that the length difference  $l_2 - l_1$  is near  $90^\circ$  at 4.2 GHz and  $l_3 - l_1$  is near  $90^\circ$  at 4.9 GHz so the error can be suppressed at the two edges of the frequency scanning range [3]. For the cases of quasi-TEM modes, the similar concept to choose the set of length is used. The line length  $l = 8, 17, 25$ mm which are denoted as  $l_1, l_2, l_3$  is used to calculate the propagation constant of the quasi-TEM mode and the line pair  $l_3 - l_2, l_2 - l_1$  is used.

#### IV. THE SIMULATED RESULTS

In order to validate the proposed method, a microstrip line with its width = 15 mm using FR4 substrate with its complex permittivity  $\epsilon_r = 4.4(1 - 0.02j)$  and substrate height  $h = 1.6$  mm is taken as an example. To be sure the simulated structure is operating the mode of our interest, the electric field on the transverse plane of the microstrip when the symmetry plane is PMC and PEC is plotted in Fig. 4 and 5, respectively. It is now evident that when the symmetry plane is used, the mode of interest that satisfies the boundary condition of the symmetry plane can be excited and the unwanted mode of the microstrip can be suppressed, which implies the single mode operation assumption can be used and proves the validity of the later calculations. Fig. 6, 7 shows the extracted normalized complex propagation constant of the quasi-TEM mode and the 1st higher order mode, respectively. The  $k_0$  is the wavenumber in free space. The formula in [5] and the Wiener-Hopf approximation used in [3] are used to compare the calculated propagation constant of the quasi TEM mode and 1st higher order mode, respectively. The well agreement of the results proves the method to be very accurate.

To further verify the calculated results, the simulated z component distribution of the electric fields for both of the modes at the edge of the microstrip along the x direction are plotted in Fig. 8, 9. Due to the limited space, only the field at 4.5 GHz is showed for demonstration. One can examined how much precise of the extracted complex propagation constant is achieved by observing the electric field distribution. One

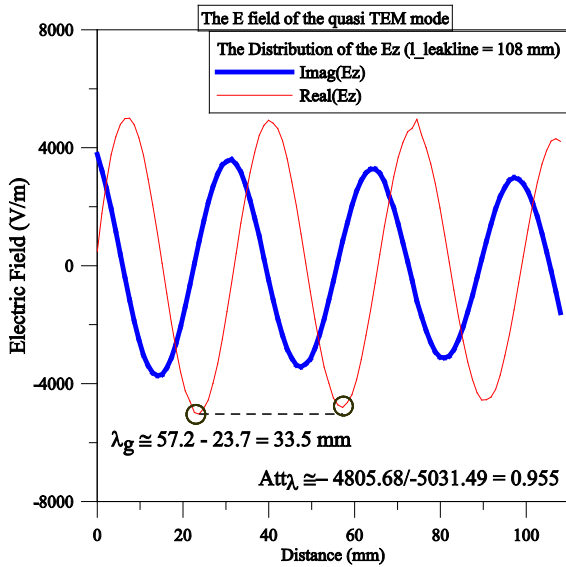


Fig. 8 The electric field of the quasi TEM mode

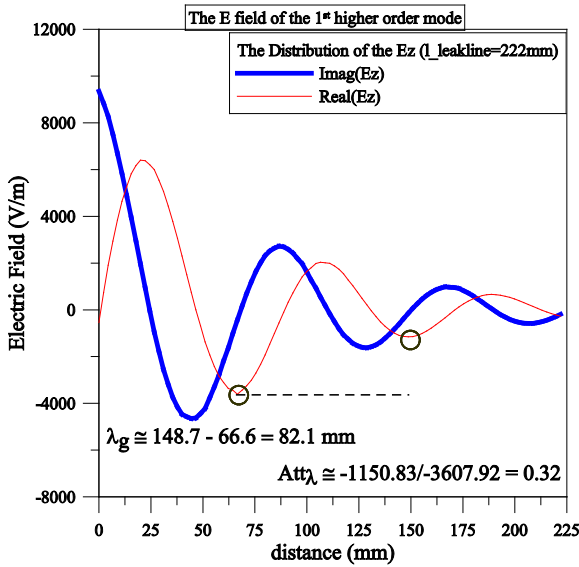


Fig. 9 The electric field of the 1st higher order mode

can first examine the extracted propagation constant in Fig. 6, 7 are  $\gamma_{TEM} / k_0 = (\alpha_{TEM} / k_0 + j\beta_{TEM} / k_0) \approx 0.0199 + j2.008$  and  $\gamma_{Leaky} / k_0 = (\alpha_{Leaky} / k_0 + j\beta_{Leaky} / k_0) \approx 0.131 + j0.764$ , respectively. The wavelength of the mode can be determined first by calculating

$$\lambda_g = \frac{C_0 k_0}{f \beta} \quad (12)$$

where  $C_0$  is the speed of light in vacuum and one can expect after the wave travels a full wavelength, the magnitude of the mode may suffer attenuation  $\hat{=} Att_\lambda$  about

$$Att_\lambda = e^{-\frac{\alpha}{k_0} \frac{2\pi k_0}{\beta}} \quad (13)$$

Equation (12) and (13) is used to estimate the wavelength and the attenuation after the wave travels a full wavelength for a quasi TEM mode and a 1<sup>st</sup> higher order mode to be

$\lambda_{g,TEM} \approx 33.2 \text{ mm}$ ,  $Att_{\lambda,TEM} = 0.94$  and  $\lambda_{g,TE10} \approx 87.2 \text{ mm}$ ,  $Att_{\lambda,TE10} = 0.34$ , respectively. One can also observe the wave length and the attenuation in a wave length in Fig. 8, 9 by finding the locations of the peaks (valleys) of the field strength and the ratio of magnitude between the first and successive peaks (valleys), as calculated in Fig. 8, 9. The calculated wave length and the attenuation per wavelength are in a fair agreement with the observation in Fig. 8, 9, indicating again that the estimated result of the propagation constant is very accurate. The above mentioned observations verifies the accuracy of the proposed method to estimate the complex propagation constant of both the quasi TEM mode and the 1<sup>st</sup> higher order mode, which can be useful in the design of microstrip leaky wave antennas.

## V. CONCLUSION

This paper propose a simple and efficient method to estimate the propagation constant of a microstrip line by using full-wave simulators with proper setup of symmetry boundary conditions and the multiline method to give an optimum estimate of the propagation constant. The simulated results are being examined and a fair agreement comparing with the theoretical estimations can be observed. The proposed method can useful in the design of microstrip leaky wave antennas which an accurate estimation of the propagation constant for both the quasi-TEM and the 1<sup>st</sup> higher order mode is needed.

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