Optimal User Group Size for Fixed Channel Allocation in Multicast OFDM Systems

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***Abstract*—Broadcast/multicast services were proposed to enhance bandwidth efficiency for multimedia streaming services. Although dynamic channel allocation can achieve better system throughput, it suffers from extremely high real-time computational complexity and a heavy communication load. By contrast, fixed channel allocation is an attractive and feasible approach, which significantly relieves the limitations. In this work, we investigate the optimal size of a multicast group that maximizes the average overall system throughput for a multicast OFDM system based on fixed channel allocation. The optimal group size can be determined beforehand and only limited real-time computation is required. We prove that finding the optimal group size can be modeled as a standard convex optimization problem.**

I. Introduction

In recent years, the dramatic growth in multimedia streaming services has increased the demand for bandwidth in wireless communications systems. Because of the broadcast nature of wireless transmission, multiple interested receivers can share the same transmission and acquire the desired service simultaneously, thereby reducing the consumption of radio resources. In broadcast/multicast OFDM systems, the achievable data rate on each channel is usually determined by the receiver suffering the worst channel condition. The objective is to ensure that all receivers allocated the channel can successfully demodulate the transmitted data. This approach is called the lowest channel gain (LCG) scheme.

Numerous radio resource allocation schemes have been proposed to enhance the overall system throughput and cope with channel variations. For example, Suh and Mo [1] developed a practical method for subcarrier allocation and rate assignment in multicast OFDM systems. Ngo et al. [2] exploited the spectrum-sharing mechanism and proposed three efficient suboptimal schemes for throughput optimization in OFDMA multicast systems. Xu et al. [3] introduced adaptive schemes for resource allocation in multicast MIMO-OFDM systems; and Demarez et al. [4], proposed adaptive bit-loading schemes for multicast OFDM systems. All of the above-mentioned works are based on the dynamic channel allocation approach. *Although dynamic channel allocation can achieve better system performance, it suffers from extremely high real-time computational complexity and a heavy communication load, which make it very hard to be implemented in practical OFDM systems involving large numbers of subcarriers and users. By contrast, fixed channel allocation relieves the limitations, and thus can be adopted in practical OFDM systems easily.*

Since our goal is to propose a practical, low-complexity solution for multicast OFDM systems, we focus only on fixed channel allocation, by which a channel is allocated to a fixed group of users requesting the same information from the serving base station (BS) and the BS adapts the transmission rate to the channel conditions of all corresponding users. Conceptually, allocating a channel to more receivers should improve the channel's overall throughput, which is defined as the total number of bits successfully decoded by all the users. However, in a fading propagation environment, the achieved data rate is inversely proportional to the user group size based on the LCG scheme, where the user group size is defined as the number of users allocated a channel. Hence, determining the optimal user group size is crucial to maximizing the average overall system throughput. Liu et al. [5], investigated the problem of finding the optimal user group size that would maximize the average system throughput, which is measured based on the channel capacity. The authors concluded that the average throughput increases with the user group size. In other words, the system should group together as many users as possible when they request the same data from the BS.

In this work, we seek the optimal user group size that maximize the average system throughput under using practical discrete data rates. Specifically, we assume that standard M-QAM modulation schemes are applied, namely, BPSK, QPSK, 16-QAM and 64-QAM, which transmit data at 1, 2, 4 and 6 bits per symbol respectively. Through our analysis, we reach a completely different conclusion to that reported in [5]; that is, grouping as many users as possible is not the best way to maximize the average overall system throughput. In fact there is an optimal user group size, and we prove that it can be found by using some convex optimization approaches.

The remainder of this paper is organized as follows. Section II describes the system model and the achievable transmission rate approximation. In Section III, we discuss the proposed approach for finding the optimal user group size for throughput maximization. The theoretical and simulation results are presented in Section IV. We then summarize our conclusion in Section V.

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This work was supported in part by the National Science Council, Taiwan, R.O.C., under Grant NSC 101-2628-E-007-014-MY2.

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DOI:10.6159/IJSE.2014.(4-1).94

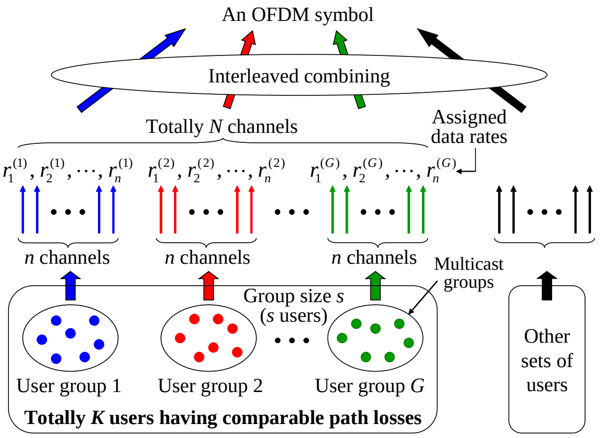

Fig. 1. Model of a multimedia multicast system with fixed channel allocation

II. Preliminaries

A. System and Channel Models

In a multicast multimedia system, the interested users in a cell may widely spread over the entire coverage area. The propagation distance between the serving BS and an interested user distributes from several meters to several kilometers. As a result, the difference of the path losses experienced by any two users could be up to 50 dB. If users experiencing large discrepancies in the path loss are clustered in a multicast group, the transmission rate on a channel will be strictly limited by the user with the worst path loss. Hence, in the view point of throughput maximization, only the users having comparable path losses should be possibly included in the same multicast group. *In the following, we focus on the scenario considering a set of interested users with comparable path losses.* The results obtained in this work can be easily extended to the scenario with multiple sets of users.

In the considered multicast OFDM system, N subcarriers are shared by K users that have comparable path losses and request the same multicast content from a BS based on fixed channel allocation. As shown in Fig. 1, the users are divided equally into G multicast groups, which are assigned the same frequency and power resource. The user group size is denoted by $s = K/G$ and the number of subcarriers assigned to a group is $n = N/G$. Furthermore, it is assumed that there are η levels of modulation with b_i bits per symbol, for $i = 1, \cdots, \eta$, in ascending order, where b_i is an integer value. For example, if BPSK, QPSK, 16-QAM and 64-QAM are applied, we have $\eta = 4$, $b_1 = 1$, $b_2 = 2$, $b_3 = 4$ and $b_4 = 6$.

We assume that the propagation channel between the BS and a receiver is a stationary Rayleigh fading channel, i.e., the power gain of the signal received on a subcarrier is exponentially distributed with unity mean. Since the K users have comparable path losses, we assume that different users experience independent and identically distributed (i.i.d.) fading channels, and the fading gains on different subcarriers are independent for each user. Consequently, we assume equal power allocation on each subcarrier to simplify the investigation on optimal user group size.

B. Achievable Transmission Rate Approximation

Let $\Omega_j^{(g)}$ denote the subcarrier index of the j-th subcarrier allocated to the g-th user group, and let $\gamma_{j,m}^{(g)}$ be the channel power gain corresponding to the m-th user on subcarrier $\Omega_j^{(g)}$ for $j = 1, \cdots, n$, $g = 1, \cdots, G$ and $m = 1, \cdots, s$. We define the average received signal-to-noise power ratio (SNR) on a subcarrier as ρ for each user. Based on the generic form for bit error rate (BER) approximation proposed in [6], the achievable data rate on subcarrier $\Omega_j^{(g)}$ can be approximated as follows:

$$r_j^{(g)} = \frac{1}{c_3} \log_2 \left(c_4 - \frac{c_2 \rho \bar{\gamma}_j^{(g)}}{\ln \left(P_{e,t}/c_1 \right)} \right), \tag{1}$$

where c_1, c_2, c_3 and c_4 are constants; $P_{e,t}$ is the pre-determined target BER at receivers; and $\bar{\gamma}_j^{(g)}$ is the channel power gain of the *index user* that has the worst channel condition among all the corresponding users, i.e., $\bar{\gamma}_j^{(g)} = \min_m \gamma_{j,m}^{(g)}$. In the considered M-QAM modulation schemes, the parameter settings $c_1 = 0.2$, $c_2 = 1.6$, $c_3 = 1$ and $c_4 = 1$ yield good approximations [6].

For the user group g, the total throughput received by the s users in an OFDM symbol is defined as $R_g = s \sum_{j=1}^{n} r_j^{(g)}$. Hence, the overall system throughput is

$$R_T = \sum_{g=1}^{G} R_g = \sum_{g=1}^{G} s \sum_{j=1}^{n} r_j^{(g)}, \tag{2}$$

which is a random variable depending on all the instantaneous channel power gains $\gamma_{j,m}^{(g)}$, and the user group size s.

III. Optimal User Group Size

A. Average Overall System Throughput

Under the assumption that the propagation channels of all users are i.i.d. Rayleigh faded with an average SNR ρ, the average overall system throughput is expressed as

$$\bar{R}_T = E \left[\sum_{g=1}^{G} s \sum_{j=1}^{n} \sum_{i=1}^{\eta} b_i \times \Pr \left(r_j^{(g)} = b_i \,|s \right) \right], \tag{3}$$

where $\Pr(r_j^{(g)} = b_i \,|s\,)$ is the probability that the transmission rate $r_j^{(g)}$ is equal to b_i, given a specific s. Since $r_j^{(g)}$ depends on the *index channel power gain* $\bar{\gamma}_j^{(g)} = \min_m \gamma_{j,m}^{(g)}$, we have

$$\Pr \left(r_j^{(g)} = b_i \,|s \right) = \int_{\alpha_i}^{\alpha_{i+1}} f_{\bar{\gamma}_j^{(g)}} \left(\gamma \right) \, d\gamma, \tag{4}$$

where $f_{\bar{\gamma}_j^{(g)}} \left(\gamma \right)$ is the distribution of $\bar{\gamma}_j^{(g)}$, which depends on the user group size s; and α_i is the threshold at which $\bar{\gamma}_j^{(g)}$ can just support the rate b_i under the target BER constraint $P_{e,t}$. Note that the order of the thresholds is as follows: $\alpha_1 <$

$\alpha_2 < \cdots < \alpha_\eta$. Moreover, we define $\alpha_{\eta+1} \equiv \infty$. According to (1), the threshold of $\bar{\gamma}_j^{(g)}$ required to support the rate b_i is

$$\alpha_i = \left(c_4 - 2^{c_3 b_i}\right) \times \ln\left(P_{e,t}/c_1\right)/c_2\rho. \quad (5)$$

In (4), $f_{\bar{\gamma}_j^{(g)}}(\gamma)$ must be determined before evaluating $\bar{R}_T$. The cumulative distribution function (CDF) of $\gamma_{j,m}^{(g)}$ is

$$F_{\gamma_{j,m}^{(g)}}(\gamma) = \Pr\left(\gamma_{j,m}^{(g)} < \gamma\right) = \begin{cases} 1 - e^{-\gamma}, & \gamma \geq 0 \\ 0, & \gamma < 0 \end{cases}. \quad (6)$$

Since $\bar{\gamma}_j^{(g)}$ is the minimum value in the set $\left\{\gamma_{j,m}^{(g)}\right\}_{m=1}^{s}$, the complementary CDF of $\bar{\gamma}_j^{(g)}$ can be expressed as

$$\tilde{F}_{\bar{\gamma}_j^{(g)}}(\gamma) = \prod_{m=1}^{s} \Pr\left(\gamma_{j,m}^{(g)} > \gamma\right) = \begin{cases} e^{-s\gamma}, & \gamma \geq 0 \\ 1, & \gamma < 0 \end{cases}. \quad (7)$$

By differentiating the CDF of $\bar{\gamma}_j^{(g)}$, we obtain the probability density function (pdf) of $\bar{\gamma}_j^{(g)}$ as

$$f_{\bar{\gamma}_j^{(g)}}(\gamma) = \begin{cases} s e^{-s\gamma}, & \gamma \geq 0 \\ 0, & \gamma < 0 \end{cases}. \quad (8)$$

Substituting (4) and (8) into (3), $\bar{R}_T$ is expressed as

$$\bar{R}_T = G\, n\, s \times \sum_{i=1}^{\eta} \left(b_i - b_{i-1}\right) e^{-\alpha_i s}, \quad (9)$$

where $b_0 \equiv 0$. Note that $\bar{R}_T$ is a function of s, i.e., $\bar{R}_T(s)$. In addition, the probability that $\bar{\gamma}_j^{(g)}$ can support the rate b_i is

$$\begin{aligned} \Pr\left(r_j^{(g)} = b_i \,|s\right) &= \Pr\left(\alpha_i \leq \bar{\gamma}_j^{(g)} < \alpha_{i+1}\right) \\ &= e^{-s\alpha_i} - e^{-s\alpha_{i+1}}. \end{aligned} \quad (10)$$

B. User Group Size Optimization

Our objective is to find the optimal user group size s that maximizes the average overall system throughput $\bar{R}_T(s)$. Hence, the optimization problem is formulated as follows:

$$\begin{aligned} &\max_s\ \bar{R}_T(s) = \max_s\ G\, n\, s \times \sum_{i=1}^{\eta} \left(b_i - b_{i-1}\right) e^{-\alpha_i s} \\ &\text{s.t.}\quad 1 \leq s \leq K, \quad P_e \leq P_{e,t}. \end{aligned} \quad (11)$$

Taking the first derivative of the objective function (9) w.r.t. s and setting it to zero, we have

$$G\, n \times \sum_{i=1}^{\eta} \left(1 - \alpha_i s\right)\left(b_i - b_{i-1}\right) e^{-\alpha_i s} = 0. \quad (12)$$

Although it is difficult to obtain a closed form solution of (12), the optimization problem can be solved by the following theorems.

Theorem 1: By applying the standard M-QAM modulation schemes, including BPSK, QPSK and other high order M-QAM, the objective function (9) is concave w.r.t. the user group size s in the range $1 \leq s \leq 1/\alpha_1$.

Proof: The second derivative of $\bar{R}_T(s)$ in (9) is

$$\bar{R}_T''(s) = G\, n \times \sum_{i=1}^{\eta} \left(\alpha_i^2 s - 2\alpha_i\right)\left(b_i - b_{i-1}\right) e^{-\alpha_i s}. \quad (13)$$

For $s = 1/\alpha_1$, we obtain

$$\bar{R}_T''(s = \tfrac{1}{\alpha_1}) = G\, n\, \alpha_1 \times \left[-b_1 e^{-1} + \sum_{i=2}^{\eta} (b_i - b_{i-1}) \Phi\left(\tfrac{\alpha_i}{\alpha_1}\right)\right], \quad (14)$$

where $\Phi(x) \triangleq \left(x^2 - 2x\right) e^{-x}$. Based on the channel capacity, increasing the number of transmission bits by one requires at least doubling the received SNR under the same bandwidth constraint; that is $\alpha_i \geq 2^{b_i - b_{i-1}} \times \alpha_{i-1}$, $\forall i$. This implies that $(b_i - b_{i-1})\,\Phi(\alpha_i/\alpha_1)$ is a monotonically decreasing function w.r.t. α_i/α_1 for $\alpha_i/\alpha_1 \geq 4$. Hence, we rewrite (14) as follows:

$$\begin{aligned} \bar{R}_T''(s = \tfrac{1}{\alpha_1}) = G\, n\, \alpha_1 \times \Big[&-b_1 e^{-1} + (b_2 - b_1)\,\Phi\left(\tfrac{\alpha_i}{\alpha_1}\right) \\ &+ \textstyle\sum_{i=3}^{\eta} (b_i - b_{i-1})\,\Phi\left(\tfrac{\alpha_i}{\alpha_1}\right)\Big]. \end{aligned} \quad (15)$$

In the summation term of (15), the ratio of two consecutive terms, e.g., $(b_{i+1} - b_i)\,\Phi\left(\frac{\alpha_{i+1}}{\alpha_1}\right)$ and $(b_i - b_{i-1})\,\Phi\left(\frac{\alpha_i}{\alpha_1}\right)$, is

$$\tfrac{(b_{i+1} - b_i)\Phi(\alpha_{i+1}/\alpha_1)}{(b_i - b_{i-1})\Phi(\alpha_i/\alpha_1)} \leq \tfrac{\Phi(2\alpha_i/\alpha_1)}{\Phi(\alpha_i/\alpha_1)} \leq 6e^{-\alpha_i/\alpha_1} < 0.11, \quad (16)$$

where the first inequality holds because $(b_i - b_{i-1})\,\Phi(\alpha_i/\alpha_1)$ is a monotonically decreasing function w.r.t. α_i/α_1 for $\alpha_i/\alpha_1 \geq 4$; and the second and third inequalities hold because $\alpha_i/\alpha_1 \geq 4$. Thus, even when $\eta \to \infty$, the summation term in (15) is still bounded by

$$\sum_{i=3}^{\eta} (b_i - b_{i-1})\,\Phi\left(\frac{\alpha_i}{\alpha_1}\right) < 1.13\,(b_3 - b_2)\,\Phi\left(\frac{\alpha_i}{\alpha_1}\right). \quad (17)$$

Substituting (17), $\alpha_2/\alpha_1 = 2$, $b_1 = 1$ and $b_2 = 2$ into (15), we obtain

$$\begin{aligned} \bar{R}_T''(s = \tfrac{1}{\alpha_1}) &< G\, n\, \alpha_1 \times \left[-e^{-1} + 1.13 \times 8e^{-4}\right] \\ &= -0.202 \times G\, n\, \alpha_1 < 0. \end{aligned} \quad (18)$$

On the other hand, the third derivative of (9) is given by

$$\bar{R}_T'''(s) = G\, n \times \sum_{i=1}^{\eta} \alpha_i^2 \left(3 - \alpha_i s\right)\left(b_i - b_{i-1}\right) e^{-\alpha_i s}. \quad (19)$$

Based on the above discussion, we can conclude that $\bar{R}_T'''(s) > 0$ for $1 \leq s \leq 1/\alpha_1$. As a result, because $\bar{R}_T''(1/\alpha_1) < 0$ and $\bar{R}_T'''(s) > 0$ for $1 \leq s \leq 1/\alpha_1$, we conclude that $\bar{R}_T(s)$ is a concave function w.r.t. s in the range $1 \leq s \leq 1/\alpha_1$. ■

Theorem 2: By applying the standard M-QAM modulation schemes, including BPSK, QPSK and other high order M-QAM, the objective function (9) is uni-modal with only one local maximum, which is in the range $1 \leq s \leq 1/\alpha_1$.

Proof: According to (11), the first derivative of (9) is

$$\bar{R}_T'(s) = G\, n \times \sum_{i=1}^{\eta} \left(1 - \alpha_i s\right)\left(b_i - b_{i-1}\right) e^{-\alpha_i s}. \quad (20)$$

Because $\alpha_{i+1} > \alpha_i$, $\forall i$, we have $\alpha_i s > 1$ for $s > 1/\alpha_1$. Hence, each term in the summation operation of (20) is negative for $s > 1/\alpha_1$, i.e., $\bar{R}_T'(s) < 0$ for $s > 1/\alpha_1$. Furthermore, based on Theorem 1, $\bar{R}_T(s)$ is a concave function in the range $1 \leq s \leq 1/\alpha_1$. Thus, we conclude that there is only one local maximum of $\bar{R}_T(s)$, which is also the global maximum. ■

Based on Theorem 1 and Theorem 2, we conclude that the global maximum of $\bar{R}_T(s)$ is in the range $1 \leq s \leq 1/\alpha_1$. Hence, we can use some standard convex/concave optimization approaches to derive the optimal user group size s^*. Substituting s^* into (9), the optimal average throughput is

$$\bar{R}_T^* = G\, n\, s^* \times \sum_{i=1}^{\eta} \left(b_i - b_{i-1}\right) e^{-\alpha_i s^*}. \quad (21)$$

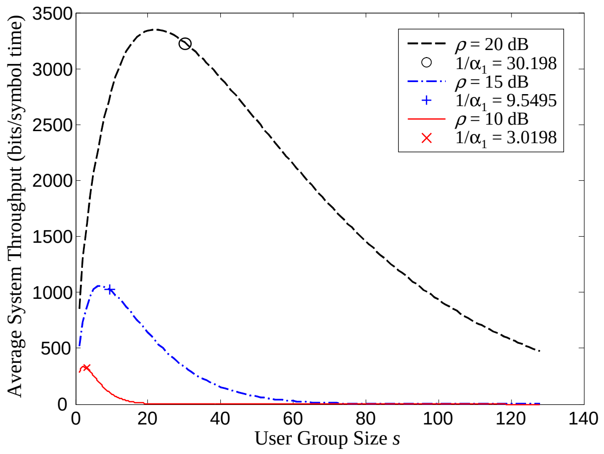

Fig. 2. Average system throughput $\bar{R}_T(s)$ versus user group size s

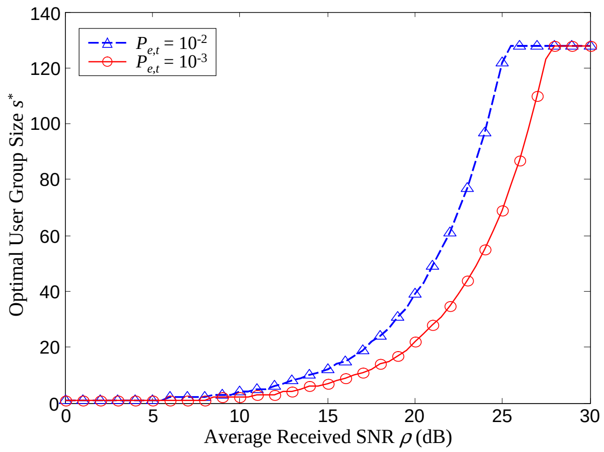

Fig. 3. Optimal user group size s^* versus the average received SNR ρ

IV. Numerical Results

Using Newtons method, we conduct the following numerical results to evaluate the performance of the proposed approach. The applied modulation schemes are BPSK, QPSK, 16-OAM and 64-QAM, i.e., $r_j \in \{1, 2, 4, 6\}$. The used parameters are $N = 256$, $K = 128$, $c_1 = 0.2$, $c_2 = 1.6$, $c_3 = 1$ and $c_4 = 1$.

Fig. 2 shows $\bar{R}_T(s)$ versus s for $\rho = 10$ dB, 15 dB and 20 dB. Obviously, the increase in the average received SNR improves $\bar{R}_T(s)$. As expected, $\bar{R}_T(s)$ is a uni-modal function with only one local maximum in the range $1 \le s \le 1/\alpha_1$, where $1/\alpha_1 = 3.0198$, 9.5495 and 30.198 for $\rho = 10$ dB, 15 dB and 20 dB respectively.

Fig. 3 shows the optimal user group size s^* versus ρ for $P_{e,t} = 10^{-2}$ and 10^{-3}. If ρ is small, s^* tends to be a small value in order to ensure that each channel can achieve an effective transmission. By contrast, if ρ is large enough, s^* is set at a large value to maximize system throughput. When ρ is very large, $s^* = 128$, which is the maximal possible value for $K = 128$. The constraint $P_{e,t} = 10^{-2}$ yields a larger optimal user group size than $P_{e,t} = 10^{-3}$, because it is more relaxed.

The theoretical $\bar{R}_T^*$ achieved by using s^* is shown in

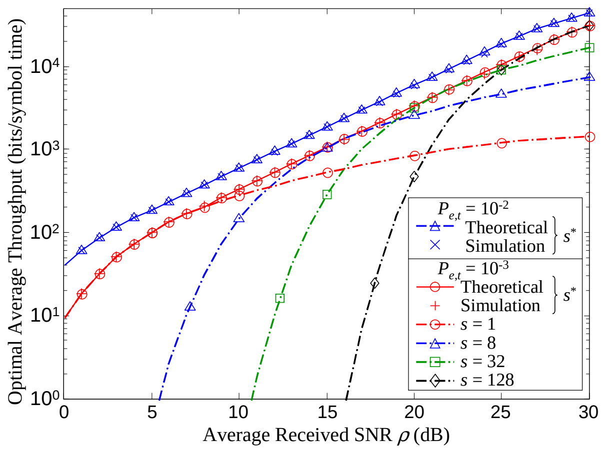

Fig. 4. Overall system throughput under the optimal user group size s^*

Fig. 4, and verified by comparing with the simulation results for $P_{e,t} = 10^{-2}$ and 10^{-3}. In the simulation, the channel gains are generated randomly and independently for different users on different channels. The simulation results, which are obtained by taking the average over 5000 rounds, fit in with the theoretical results very well. In comparison with the results derived by using a fixed user group size, the proposed approach always yields the best overall system throughput.

V. Conclusion

In view of the rapid growth in the demand for multimedia services, we study the multicast user group size optimization problem for throughput maximization in a fixed channel allocation scenario. By applying discrete rate modulation, such as the M-QAM schemes, we find that there is an optimal user group size that maximizes the average system throughput. The optimization problem can be solved easily by using standard convex optimization approaches. The results reported in this work are based on practical considerations and can serve as the design guidelines for resource allocation and user scheduling of multimedia multicast services.

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