Performance of Fixed Gain Amplify-and-Forward Relay Systems with Partial Channel State Information

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Abstract— **For amplify-and-forward (AF) relay systems employing coherent modulation, the receiver at the destination requires the full channel state information (CSI) of the source-relay-destination (SRD) links and the amplified relay-destination (RD) links to perform maximal ratio combining (MRC). To reduce the burden of channel estimation and CSI forwarding in AF relay systems, we propose an MRC receiver with partial CSI which needs the CSI of the SRD links and the variance of the source-relay (SR) links. We analyze the bit-error-rate (BER) of the proposed scheme based on the moment generating function approach. Compared with the full CSI case, the BER performance degradation due to partial CSI depends on the channel variances of the SR and RD links. When the channel variances of the SR links are much larger than those of the RD links, the performance loss due to partial CSI is negligible.**

I. INTRODUCTION

The employment of relays in cellular and ad hoc networks can not only increase system capacity but also extend coverage area [1]. In this letter, we consider an ad hoc relay network in which the signal transmitted from the source is amplified and forwarded by multiple relays to the destination. It has been shown the amplify-and-forward (AF) relaying protocol can achieve full diversity gain with much less computing power as no decoding is performed at the relay [2].

When coherent modulation is employed for signal transmission over fading channels, the receiver of a diversity system must acquire channel state information (CSI) to perform maximal ratio combining (MRC). Previous works studying the channel estimation problem in AF relay networks focus on how to estimate the composite channel gain of the source-relay-destination (SRD) link [3]-[5]. Since the noise at the relay is amplified and forwarded to the destination, the instantaneous noise variances of different SRD links at the destination are not the same and need to be estimated first and then normalized to perform MRC [6]. The instantaneous noise variance estimation and its effect on the performance of AF relay networks have been discussed in [7].

When the number of relays or the number of antennas at the relay is large, the estimation of CSI and instantaneous noise variance becomes more complicated. To reduce the burden of instantaneous noise variance estimation, we propose an MRC receiver with partial CSI which only requires the CSI of the SRD links and the variances of the source-relay (SR) links. Furthermore, we derive the theoretical bit-error-rate (BER) formula and study how the values of channel variances affect the BER performance of the proposed scheme.

Notation: Symbols for matrices and vectors are in boldface. $(\cdot)^{\mathsf{T}}$ stands for transpose and $(\cdot)^{\mathsf{H}}$ denotes Hermitian transpose.

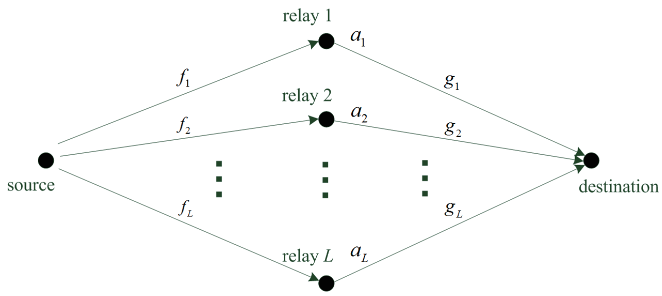

Fig. 1. Diagram of a dual-hop, multiple relay system.

$\Re(z)$ and $\Im(z)$ represent the real part and imaginary part of a complex number z, respectively. $X \sim \mathcal{CN}(\mu, \sigma^2)$ means the random variable X is a circularly symmetric complex Gaussian random variable with mean μ and variance σ^2. We use the symbols $\det(\cdot)$, $|\cdot|$, $*$, and $\mathbb{E}[\cdot]$ to denote the determinant, the Euclidean norm, the complex conjugation, and the expectation operator, respectively.

II. SYSTEM MODEL

The dual-hop AF relay system with one source, multiple relays, and one destination is depicted in Fig. 1. We assume the source, relays, and destination are all mobile devices with single antenna for signal transmission and reception. During the first time interval, the source broadcasts a signal to the relays and the received discrete-time baseband signals at the relays are given by

$$r_i = \sqrt{E_s} f_i x + n_i, \ i = 1, 2, \cdots, L \tag{1}$$

where i is the index of the relay, L is the total number of relays, E_s is the average energy of the transmitted modulation symbol, and $x \in \{-1, +1\}$ is the transmitted binary phase-shift keying (BPSK) modulation symbol. The SR channel gains f_i are independently distributed from $\mathcal{CN}(0, \sigma_{f_i}^2)$ and the additive white Gaussian noises (AWGN) n_i are independent, identically distributed (i.i.d.) from $\mathcal{CN}(0, N_0)$.

After receiving the signal, the relays amplify and forward the signals r_i to the destination through orthogonal channels to avoid interfering with each other at the destination. The received signals y_i at the destination are given by

$$y_i = a_i g_i r_i + \eta_i, \ i = 1, 2, \cdots, L \tag{2}$$

where the amplification gain a_i is determined by the average

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DOI:10.6159/IJSE.2014.(4-1).97

symbol energy constraint E_s at the relay node i as [8]

$$a_i^2 = \mathbb{E}\left[\frac{E_s}{E_s|f_i|^2+N_0}\right] = \frac{1}{\sigma_{f_i}^2}\exp\left(\frac{N_0}{\sigma_{f_i}^2 E_s}\right)\Gamma\left(0,\frac{N_0}{\sigma_{f_i}^2 E_s}\right), \quad (3)$$

and $\Gamma(b,z)=\int_z^\infty t^{b-1}e^{-t}dt$ is the incomplete Gamma function. The relay-destination (RD) channel gains g_i and the AWGNs η_i are independently distributed from $\mathcal{CN}(0,\sigma_{g_i}^2)$ and $\mathcal{CN}(0,N_0)$, respectively. Finally, we assume random variables f_i, g_i, n_i, and η_i are all independent to each other.

Substituting (1) into (2), the ith received signal at the destination node can be expressed as

$$y_i = \sqrt{E_s}a_i f_i g_i x + a_i g_i n_i + \eta_i,\ i=1,2,\cdots,L \quad (4)$$

Under the assumption that $a_i g_i$ is known at the receiver, the instantaneous variance of the noise term $a_i g_i n_i + \eta_i$ in (4) is

$$\mathbb{E}[|a_i g_i n_i + \eta_i|^2] = (|a_i g_i|^2+1)N_0,\ i=1,2,\cdots,L \quad (5)$$

Since the instantaneous noise variance depends on $|a_i g_i|^2+1$, the MRC output d can be derived as the sum of the weighted received signals y_i given by [6]

$$d = \frac{\sqrt{E_s}}{N_0}\sum_{i=1}^{L}\frac{(a_i f_i g_i)^*}{(|a_i g_i|^2+1)}y_i. \quad (6)$$

For demodulating the BPSK symbol, the MRC receiver needs to know the composite SRD channel gains $a_i f_i g_i$ and the amplified RD channel gains $a_i g_i$. In practical AF relaying systems, the composite SRD channel gains $a_i f_i g_i$ can be estimated at the destination based on the received pilot symbols sent from the source [3]. The amplified RD channel gains $a_i g_i$ can be obtained in two ways. The first method is to estimate the SR channel gains f_i at the relay and forward the estimated channel gains $\hat{f}_i$ to the destination. Then the estimated amplified RD channel gains $\widehat{a_i g_i}$ can be determined by dividing the estimated composite channel gains $\widehat{a_i f_i g_i}$ by $\hat{f}_i$. When the SR channel gains are time-varying or the number of SR links is large, the continual transmission of the estimated CSI $\hat{f}_i$ from the relay to the destination will consume too much system bandwidth and power. The second method is to estimate the amplified RD channel gains $a_i g_i$ directly from the received pilot signals at the destination [7]. Since the receiver at the destination needs to estimate both $a_i f_i g_i$ and $a_i g_i$, some pilot symbols transmitted from the source are reset at the relay node for the estimation of $a_i g_i$. Therefore, the reliabilities of the estimates $\widehat{a_i f_i g_i}$ and $\widehat{a_i g_i}$ degrade since fewer effective pilot symbols are used in each estimation process.

To remedy the aforementioned drawbacks of conventional CSI and instantaneous noise variance estimation in AF relay systems, we consider to replace the instantaneous amplified channel gain $|a_i g_i|^2$ by its mean $a_i^2\sigma_{g_i}^2$ and the corresponding MRC output $\hat{d}$ is given by

$$\hat{d} = \frac{\sqrt{E_s}}{N_0}\sum_{i=1}^{L}\frac{(a_i f_i g_i)^*}{(a_i^2\sigma_{g_i}^2+1)}y_i. \quad (7)$$

For the proposed MRC receiver with partial CSI in practical AF systems, the destination takes all the received pilot symbols to estimate the composite channel gains $a_i f_i g_i$. The variances of the SR links $\sigma_{f_i}^2$ are estimated at the relay based on the pilot symbols and the estimates $\hat{\sigma}_{f_i}^2$ are forwarded to the destination for performing MRC. When the fading channels are ergodic in the interested interval of transmission, the overhead of forwarding $\hat{\sigma}_{f_i}^2$ from the relay to the destination is much less compared with forwarding the time-varying channel estimates $\hat{f}_i$. At the destination, the values of $a_i^2\sigma_{g_i}^2$ can be estimated via dividing the sample variances of $a_i f_i g_i$ by $\hat{\sigma}_{f_i}^2$.

Since we use the mean of $a_i^2 g_i^2$ to replace its instantaneous value, the resulting error can be characterized by the standard deviation of $a_i^2 g_i^2$ which is equal to $a_i^2\sigma_{g_i}^2$. Fig. 2 shows the value of a_i^2 is a decreasing function of the parameter $\sigma_{f_i}^2$. In other words, the mismatch due to partial CSI is an increasing function of the channel variance ratio $\sigma_{g_i}^2/\sigma_{f_i}^2$.

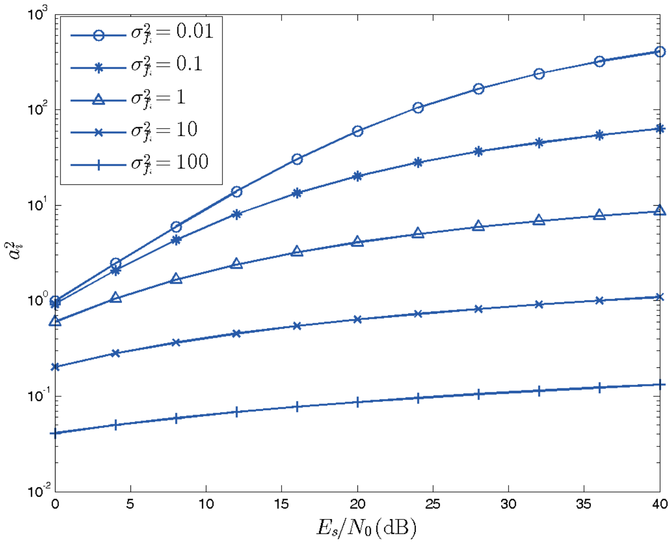

Fig. 2. a_i^2 versus E_s/N_0 for different values of $\sigma_{f_i}^2$.

III. Performance Analysis

In the section, we analyze the performance of the proposed MRC receiver with partial CSI for AF relay systems. For completeness and comparison purpose, we also present the BER formula of the MRC receiver with full CSI.

A. MRC with Full CSI

The analysis basically follows the approach outlined in [9]. Given that $a_i f_i g_i$ and $a_i g_i$ are both perfectly known at the receiver, the signal-to-noise ratio (SNR) corresponding to the MRC receiver (6) is given by

$$\gamma = \sum_{i=1}^{L}\gamma_i,\quad \gamma_i = \frac{E_s}{N_0}\frac{|a_i f_i g_i|^2}{|a_i g_i|^2+1}. \quad (8)$$

For BPSK modulation, the conditional BER corresponding to the SNR γ is

$$P_b = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{\gamma_1+\gamma_2+\cdots+\gamma_L}{\sin^2\theta}\right)d\theta \quad (9)$$

Since the channel gains in different links are independent to each other, the unconditional BER is given by

$$\overline{P}_b = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{L} M_i\left(\frac{1}{\sin^2\theta}\right)d\theta, \quad (10)$$

where $M_i(\cdot)$ is the moment generating function (MGF) of γ_i. Since $f_i \sim \mathcal{CN}(0,\sigma_{f_i}^2)$ and $g_i \sim \mathcal{CN}(0,\sigma_{g_i}^2)$, the random variables $|f_i|^2$ and $|g_i|^2$ are both exponential random variables with means $\sigma_{f_i}^2$ and $\sigma_{g_i}^2$, respectively. Therefore, the MGF $M_i(s)$ is given by

$$M_i(s) = \int_0^\infty\int_0^\infty \exp\left(-s\frac{E_s}{N_0}\frac{a_i^2uv}{a_i^2v+1}\right)\times \frac{1}{\sigma_{f_i}^2}\exp\left(-\frac{u}{\sigma_{f_i}^2}\right)\frac{1}{\sigma_{g_i}^2}\exp\left(-\frac{v}{\sigma_{g_i}^2}\right)dudv$$

$$= \frac{1}{\frac{E_s}{N_0}\sigma_{f_i}^2 s+1} + \frac{\frac{E_s}{N_0}\sigma_{f_i}^2 s}{\left(\frac{E_s}{N_0}\sigma_{f_i}^2 s+1\right)^2 a_i^2\sigma_{g_i}^2}\times \exp\left[\frac{1}{\left(\frac{E_s}{N_0}\sigma_{f_i}^2 s+1\right)a_i^2\sigma_{g_i}^2}\right]\Gamma\left(0,\frac{1}{\left(\frac{E_s}{N_0}\sigma_{f_i}^2 s+1\right)a_i^2\sigma_{g_i}^2}\right). \quad (11)$$

B. MRC with Partial CSI

When the instantaneous channel variance $(a_i^2g_i^2+1)N_0$ is replaced by $(a_i^2\sigma_{g_i}^2+1)N_0$, the SNR of the MRC output (7) becomes too complicated to do analysis directly. To evaluate the BER performance, we first consider the conditional BER given by

$$P_b = \Pr\left(\Re(\hat{d})<0|x=1,\boldsymbol{g}\right), \quad (12)$$

where $\boldsymbol{g} = [g_1\ g_2\ \cdots\ g_L]^\mathsf{T}$. Substituting (4) into (7), the decision variable $\Re(\hat{d})$ given $x=1$ is explicitly given by

$$\begin{aligned}\Re(\hat{d}) &= \sum_{i=1}^{L}\frac{E_s}{N_0}\frac{|a_if_ig_i|^2}{w_i} + \sum_{i=1}^{L}\frac{\sqrt{E_s}}{N_0}\frac{\Re\{(a_if_ig_i)^*z_i\}}{w_i} \\ &= \sum_{i=1}^{L}\left(A_i|f_i|^2 + C_if_iz_i^* + C_i^*f_i^*z_i\right), \quad (13)\end{aligned}$$

where $w_i = a_i^2\sigma_{g_i}^2+1$, $z_i = a_ig_in_i+\eta_i$, $A_i = \frac{E_s}{N_0}\frac{|a_ig_i|^2}{w_i}$, and $C_i = \frac{1}{2}\frac{\sqrt{E_s}}{N_0}\frac{a_ig_i}{w_i}$. To facilitate later discussion, we express the decision variable $\Re(\hat{d})$ in matrix form as

$$\Re(\hat{d}) = \sum_{i=1}^{L}\boldsymbol{v}_i^\mathsf{H}\boldsymbol{Q}_i\boldsymbol{v}_i, \quad (14)$$

where $\boldsymbol{v}_i = [f_i\ z_i]^\mathsf{T}$ and $\boldsymbol{Q}_i = \begin{bmatrix} A_i & C_i^* \\ C_i & 0\end{bmatrix}$. Since the mean vector of $\boldsymbol{v}_i$ is $\mathbf{0}$, the covariance matrix $\boldsymbol{L}_i$ of the vector $\boldsymbol{v}_i$ is

$$\boldsymbol{L}_i = \begin{bmatrix}\sigma_{f_i}^2 & 0 \\ 0 & \sigma_{z_i}^2\end{bmatrix}, \quad (15)$$

where $\sigma_{z_i}^2 = (a_i^2|g_i|^2+1)N_0$ is the conditional variance of z_i.

Since the decision variable $\Re(\hat{d})$ is in the Hermitian quadratic form of complex Gaussian random variables, its conditional characteristic function $\phi_{\boldsymbol{g}}(s)$ can be computed by using the main theorem given in [10] as

$$\begin{aligned}\phi_{\boldsymbol{g}}(s) &= \prod_{i=1}^{L}\frac{1}{\det(\boldsymbol{I}_2+s\boldsymbol{L}_i\boldsymbol{Q}_i)} \\ &= \prod_{i=1}^{L}\frac{1}{1+s\frac{E_s}{N_0}\frac{a_i^2\sigma_{f_i}^2|g_i|^2}{w_i} - \frac{s^2}{4}\frac{E_s}{N_0}\frac{a_i^2\sigma_{f_i}^2|g_i|^2(a_i^2|g_i|^2+1)}{w_i^2}}, \quad (16)\end{aligned}$$

where $\boldsymbol{I}_2$ denotes the 2×2 identity matrix. Since $|g_i|^2$ are independent exponential random variables with means $\sigma_{g_i}^2$, the unconditional characteristic function of $\Re(\hat{d})$ is

$$\phi(s) = \prod_{i=1}^{L}\int_0^\infty \frac{\frac{1}{\sigma_{g_i}^2}e^{-v/\sigma_{g_i}^2}}{1+s\frac{E_s}{N_0}\frac{a_i^2\sigma_{f_i}^2v}{w_i} - \frac{s^2}{4}\frac{E_s}{N_0}\frac{a_i^2\sigma_{f_i}^2v(a_i^2v+1)}{w_i^2}}dv. \quad (17)$$

Finally, the unconditional BER $\widetilde{P}_b$ can be calculated based on Gauss-Chebyshev quadratures [11] as

$$\begin{aligned}\widetilde{P}_b &= \Pr\left(\Re(\hat{d})<0|x=1\right) \\ &= \frac{1}{N}\sum_{k=1}^{N/2}\{\Re[\phi(c+jc\tau_k)] + \tau_k\Im[\phi(c+jc\tau_k)]\} + e_N, \quad (18)\end{aligned}$$

where $\tau_k = \tan((2k-1)\pi/(2N))$ and e_N is an error term that tends to zero as N goes to infinity. Moreover, c is a positive real number chosen to guarantee quick convergence. From our own numerical experiment experiences, the values of $N=128$ and $c=0.8$ usually yield satisfactory accuracy.

IV. Numerical Results

In our numerical experiments, we assume the MRC receiver knows the composite SRD channel gains $a_if_ig_i$ perfectly. For the MRC receiver with full and partial CSI, those terms $|a_ig_i|^2$ and $a_i^2\sigma_{g_i}^2$ are available perfectly at the receiver, respectively.

Fig. 3 shows the BER performance of the MRC receiver with full and partial CSI in AF relay systems. The channel variances of SR and RD links are $\sigma_{f_i}^2 = 1$ and $\sigma_{g_i}^2 = 1$ for $i = 1,2,\cdots,L$, respectively. The theoretical results are illustrated by lines and the computer simulation results are represented by markers. It can be clearly seen the results obtained from our analysis match very well with the simulation results. When the number of antennas L is greater than 1, the SNR gains of the receiver with full CSI over the one with partial CSI are 0.8 dB and 1.2 dB for $L=2$ and $L=4$ at BER 10^{-5}, respectively.

Since the main difference between the full CSI receiver and the partial CSI receiver lies in the availability of the instantaneous channel gains g_i, we expect there will be larger performance discrepancy when the channel variances of g_i increase. Under the parameter setting $\sigma_{f_i}^2 = 1$ and $\sigma_{g_i}^2 = 10$ for $i=1,2,\cdots,L$, Fig. 4 indicates the SNR gaps between the full and partial CSI increase to 2.5 dB and 3.0 dB for $L=2$ and $L=4$ at BER 10^{-5}, respectively.

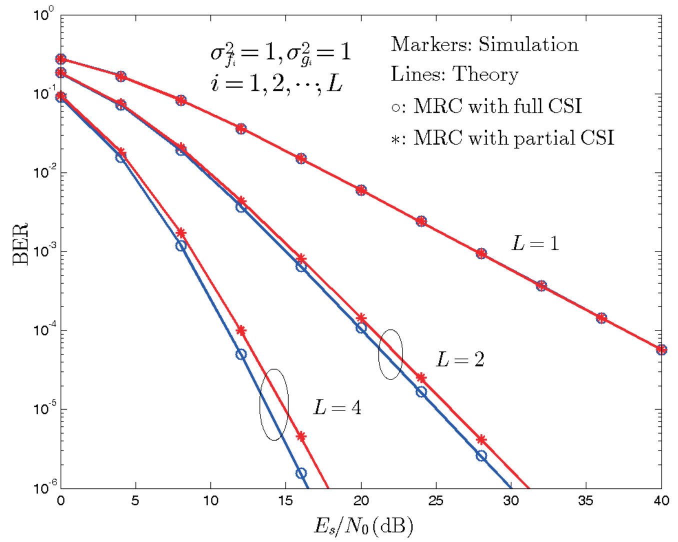

Fig. 3. BER of the MRC receiver with full and partial CSI in AF relay systems. $\sigma_{f_i}^2 = 1$ and $\sigma_{g_i}^2 = 1$ for $i = 1, 2, \cdots, L$. $\sigma_{g_i}^2/\sigma_{f_i}^2 = 1$.

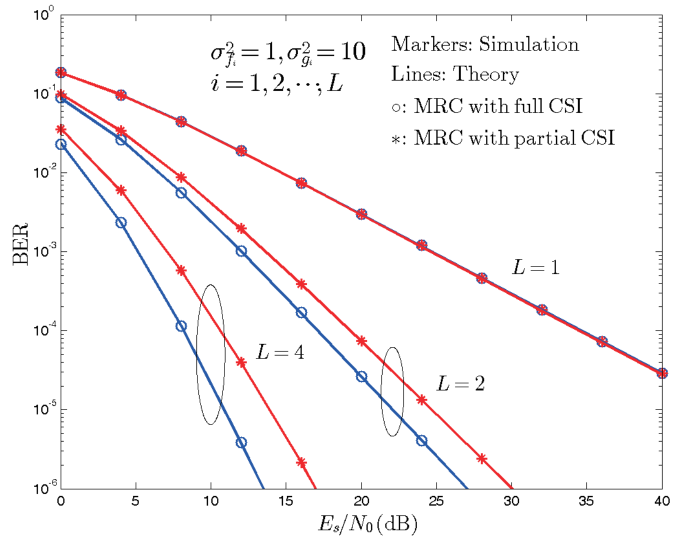

Fig. 4. BER of the MRC receiver with full and partial CSI in AF relay systems. $\sigma_{f_i}^2 = 1$ and $\sigma_{g_i}^2 = 10$ for $i = 1, 2, \cdots, L$. $\sigma_{g_i}^2/\sigma_{f_i}^2 = 10$.

Finally, Fig. 5 illustrates the BER performance of the MRC receiver when the relays are closer to the source than the destination. In this scenario, we set the channel variances $\sigma_{f_i}^2 = 10$ and $\sigma_{g_i}^2 = 1$ for $i = 1, 2, \cdots, L$. It can be seen in Fig. 4, there is almost no any SNR degradation due to the replacement of $a_i^2|g_i|^2$ by $a_i^2\sigma_{g_i}^2$ in the MRC receiver. The reason behind this phenomenon is that the larger $\sigma_{f_i}^2$ results in the smaller amplification gain a_i and the difference between $a_i^2|g_i|^2$ and $a_i^2\sigma_{g_i}^2$ is scaled down by a_i^2. Therefore, it is more desirable to select neighboring relays to cooperative, especially when the MRC receiver with partial CSI is a preferred choice.

V. Conclusions

For AF relay systems in Rayleigh fading channels, we proposed an low-complexity receiver which only requires partial CSI to perform MRC. The performance of the proposed scheme was analyzed accurately based on the MGF approach. Our analysis shows that the SNR loss due to partial CSI depends on the relative magnitude of the channel variances of the SR links and the RD links. To minimize the performance loss due to partial CSI, the position of the relay should be placed closer to the source than to the destination.

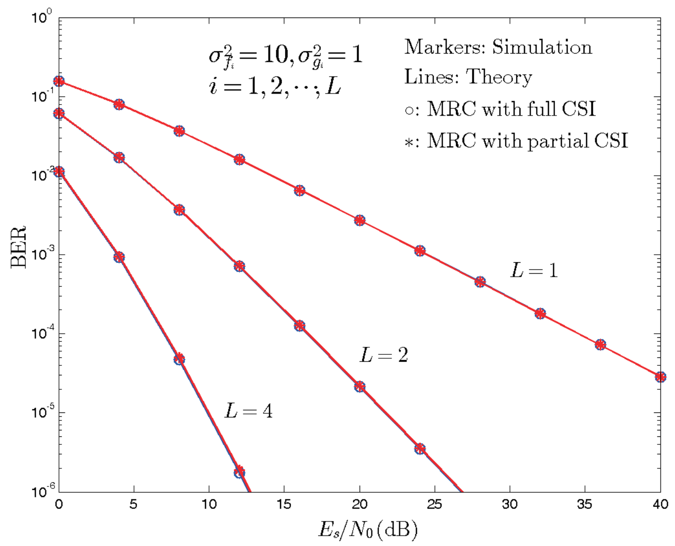

Fig. 5. BER of the MRC receiver with full and partial CSI in AF relay systems. $\sigma_{f_i}^2 = 10$ and $\sigma_{g_i}^2 = 1$ for $i = 1, 2, \cdots, L$. $\sigma_{g_i}^2/\sigma_{f_i}^2 = 0.1$.

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