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Delay-Dependant Stability of Chaotic Systems with Exponential Synchronization

F.H. Hsiao[1,*]
[1] Department of Electrical Engineering, National University of Tainan, 33, Section 2, Shu Lin Street, Tainan 70005, Taiwan, R.O.C.

Abstract* -- This study is concerned with the synchronization problem for a class of chaotic systems with multiple time-delays and transmitted noise. A dynamic compensator is proposed to achieve the synchronization between master and slave chaotic systems with multiple time-delays. First, the neural-network (NN) model is employed to approximate the chaotic systems with multiple time delays. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of the NN model. Based on the LDI state-space representation, an exponential stability criterion of error dynamics derived in terms of Lyapunov's direct method is proposed to ensure that the trajectories of the slave system can approach those of master system. Subsequently, the stability condition of this criterion is reformulated into a linear matrix inequality (LMI). Based on the LMI, a fuzzy controller is synthesized to realize the exponential synchronization of the chaotic master-slave systems. Finally, this study provides a numerical example with simulations is given to illustrate the concepts discussed throughout this work.

***Index Terms* -- Neural network (NN), Chaotic synchronization, Fuzzy control .**

I. INTRODUCTION

Chaos is a well-known nonlinear phenomenon in the world (see [1-3]). It is an irregular, seemingly random and extreme sensitivity to initial conditions [4]. On the basis of these properties, chaos has been applied in communication and there had been many successful applications many fields such as electric circuits, and so on [5-8]. One of its applications in communication, chaotic synchronization, has been investigated extensively.

The chaotic synchronization proposed by Pecora and Carroll in 1990 [9] is to control a chaotic system to follow another one. Since they introduced this conception, various synchronization approaches, such as secure communications [10], adaptive control [11], have been expanded widely for chaotic synchronization in the past two decades and chaotic synchronization has become a popular study (see [10-19]).

Neural-network (NN)-based modeling has become an active research field in the past few years because of its unique merits in solving complex nonlinear system identification and control problems [20-24]. Neural networks are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As a result, we can train an NN to represent a particular function by adjusting the weights between elements.

Fuzzy control has attracted a great deal of attention from both the academic and industrial communities during the last decade, and there have been many successful applications [25-29]. Despite the success, it has become evident that many basic issues remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. Lately, there have been significant research efforts devoted to these issues (see [30-33]).

In practice, due to the information transmission, multiple time-delays naturally exist in many systems. The existence of multiple time-delays is frequently a source of instability and encountered in various engineering systems [34-35]. In general, the introduction of multiple time-delays factor complicates the analysis, and hence convenient methods to check stability have long been sought after. The stability criteria of time-delay systems so far have been approached in two main ways according to the dependence upon the size of delay. One direction is to contrive stability conditions that do not include information on the delay, while the other direction includes methods which take time delay into account. The former case is often referred to as delay-independent criteria and generally gives good algebraic conditions. However, abandonment of information on the size of multiple time-delays necessarily causes conservativeness of the criteria, especially when the delay is comparatively small.

Therefore, a method of realizing exponential synchronization is proposed for the multiple time-delay chaotic systems via the neural-network (NN) model in this study. First, the neural-network (NN) model is employed to approximate the chaotic systems with multiple time-delays. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of the NN model. Next, in terms of Lyapunov's direct method, a delay-dependant criterion is derived to guarantee the stability of the error system between the master system and slave system. Subsequently, the stability condition of this delay-dependant criterion is reformulated into a linear matrix inequality (LMI). Based on the LMI, a fuzzy controller is synthesized to realize the exponential synchronization of the chaotic master-slave systems. Finally, a numerical example with simulations is given to illustrate the concepts discussed throughout this work.

* Corresponding author: *fhhsiao@mail.nutn.edu.tw*
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II. PRELIMINARY NOTATIONS

The following notations will be used throughout this paper.

N_d : multiple time-delay chaotic system (see (3.1)).

N_m : master system.

X : state vector of master system

N_s : slave system.

$\hat{X}$: state vector of slave system

N_e : error system between master system and slave system.

E : synchronization error ($E(t) \equiv X(t) - \hat{X}(t)$)

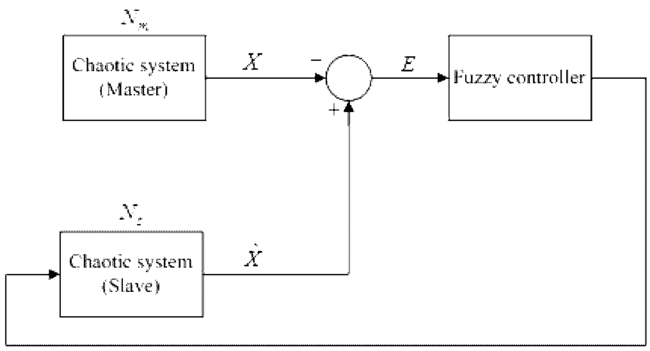

Fig. 1 Chaotic synchronization scheme

Remark 2.1: A model-based fuzzy controller is synthesized to exponentially stabilize the error system so that the trajectories of the slave system can approach those of the master system.

III. SYSTEM DESCRIPTION

Consider a multiple time-delay chaotic system described by the following equation:

$$N_d: \quad \dot{X}(t) = f(X(t)) + \sum_{k=1}^{m} H_k(X(t-\tau_k)) \qquad (3.1)$$

where $f(\cdot)$ and $H_k(\cdot)$ are the nonlinear vector-valued functions, $X(t)$ denotes the state vector, $\tau_k (k = 1, 2, \cdots, m)$ are the time delays.

In this section, an NN model is first established to approximate the chaotic system. Then, the dynamics of the NN model is converted into a linear differential inclusion (LDI) state-space representation. Finally, according to the LDI state-space representation, the fuzzy controller is synthesized to realize the synchronization of chaotic system.

A. Neural-Network (NN) Model

The chaotic system N_d is approximated by an NN model, shown in Fig. 2, has S layers with $J^{\sigma} (\sigma = 1, 2, \cdots, S)$ neurons for each layer, in which $x_1(t) \sim x_\delta(t)$ are the state variables, $x_1(t-\tau_1) \sim x_1(t-\tau_m)$, $x_2(t-\tau_1) \sim x_\delta(t-\tau_m)$ are the state variables with delays.

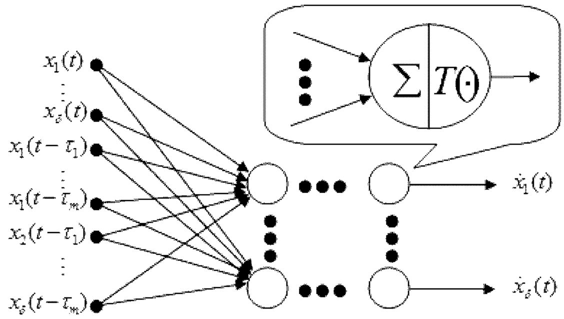

Fig. 2. An NN model for N_d

In order to distinguish among these layers, the superscripts are used for identifying the layers. Specifically, we append the number of the layer as a superscript to the names for each of these variables. Thus, the weight matrix for the σ th layer is written as W^{σ}. Moreover, it is assumed that $v_{\varsigma}^{\sigma}(t)$ $(\varsigma = 1, 2, \cdots, J^{\sigma}; \ \sigma = 1, 2, \cdots, S)$ is the net input and $T(v_{\varsigma}^{\sigma}(t))$ is the transfer function of the neuron. Subsequently, the transfer function vector of the σ th layer can be written as:

$$\Psi^{\sigma}(v_{\varsigma}^{\sigma}(t)) \equiv [T(v_1^{\sigma}(t)) \ T(v_2^{\sigma}(t)) \ \cdots \ T(v_{J^{\sigma}}^{\sigma}(t))]^T, \ \sigma = 1, 2, \cdots, S \quad (3.2)$$

where $T(v_{\varsigma}^{\sigma}(t))$ $(\varsigma = 1, 2, \cdots, J^{\sigma})$ is the transfer function of the ς th neuron. Then the final output of NN model can be inferred as follows:

$$\dot{X}(t) = \Psi^S(W^S \Psi^{S-1}(W^{S-1}\Psi^{S-2}(\cdots \Psi^2(W^2 \Psi^1(W^1 \Lambda(t)))\cdots))) \qquad (3.3)$$

where $\Lambda^T(t) = [X^T(t) \ X^T(t-\tau_k)]$

with $X(t) = [x_1(t) \ x_2(t) \ \cdots \ x_\delta(t)]^T$,

$$X(t-\tau_k) = [x_1(t-\tau_1) \cdots x_1(t-\tau_m) \ x_2(t-\tau_1) \ \cdots \ x_\delta(t-\tau_m)]^T$$

for $k = 1, 2 \cdots, m$,.

B. Linear Differential Inclusion (LDI)

In order to deal with the stability problem of the chaotic systems, an LDI state-space representation is established for the dynamics of the NN model and it can be described as [36,37]:

$$\dot{Y}(t) = A(a(t))Y(t), A(a(t)) = \sum_{i=1}^{\phi} h_i(a(t))\overline{A}_i \qquad (3.4)$$

where φ is a positive integer, $a(t)$ is a vector signifying the dependence of $h_i(\cdot)$ on its elements, $\overline{A}_i$ $(i = 1, 2, \cdots, \phi)$ are constant matrices and $Y(t) = [y_1(t) \ y_2(t) \ \cdots \ y_{\Xi}(t)]^T$.

Furthermore, it is assumed that $h_i(a(t)) \geq 0$, $\sum_{i=1}^{\phi} h_i(a(t)) = 1$. From the properties of LDI, without loss of generality, we can use $h_i(t)$ instead of $h_i(a(t))$. In the following, a procedure is taken to represent the dynamics of the NN model (3.4) by LDI state-space representation [36].

To begin with, notice that the output $T(v_\varsigma^\sigma(t))$ satisfies

$$g_{\varsigma 0}^\sigma v_\varsigma^\sigma(t) \leq T(v_\varsigma^\sigma(t)) \leq g_{\varsigma 1}^\sigma v_\varsigma^\sigma(t), \qquad v_\varsigma^\sigma(t) \geq 0$$

$$g_{\varsigma 1}^\sigma v_\varsigma^\sigma(t) \leq T(v_\varsigma^\sigma(t)) \leq g_{\varsigma 0}^\sigma v_\varsigma^\sigma(t), \qquad v_\varsigma^\sigma(t) < 0$$

where $g_{\varsigma 0}^\sigma$ and $g_{\varsigma 1}^\sigma$ denote the minimum and the maximum of the derivative of $T(v_\varsigma^\sigma(t))$, respectively, and they are given in the following:

$$g_{\varsigma\varphi}^\sigma = \begin{cases} \min\limits_{v} \dfrac{d\,T(v_\varsigma^\sigma(t))}{dv_\varsigma^\sigma(t)} & \text{when} \quad \varphi = 0 \\ \max\limits_{v} \dfrac{d\,T(v_\varsigma^\sigma(t))}{dv_\varsigma^\sigma(t)} & \text{when} \quad \varphi = 1. \end{cases} \tag{3.5}$$

Subsequently, the min-max matrix G^σ of the σ th layer is defined as follows:

$$G^\sigma \equiv diag[g_{\varsigma\varphi_\varsigma}^\sigma] = \begin{bmatrix} g_{1\varphi_1}^\sigma & 0 & 0 & \cdots & 0 \\ 0 & g_{2\varphi_2}^\sigma & 0 & \ddots & 0 \\ 0 & 0 & g_{3\varphi_3}^\sigma & 0 & \vdots \\ \vdots & \ddots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & g_{J^\sigma \varphi_J}^\sigma \end{bmatrix}. \tag{3.6}$$

Moreover, based on the interpolation method, the transfer function $T(v_\varsigma^\sigma(t))$ can be represented as follows [36]:

$$\begin{aligned} T(v_\varsigma^\sigma(t)) &= (h_{\varsigma 0}^\sigma(t) g_{\varsigma 0}^\sigma + h_{\varsigma 1}^\sigma(t) g_{\varsigma 1}^\sigma) v_\varsigma^\sigma(t) \\ &= (\sum_{\phi=0}^{1} h_{\varsigma\phi}^\sigma(t) g_{\varsigma\phi}^\sigma) v_\varsigma^\sigma(t) \end{aligned} \tag{3.7}$$

where the interpolation coefficients $h_{\varsigma\phi}^\sigma(t) \in [0,\ 1]$ and $\sum_{\phi=1}^{2} h_{\varsigma\phi}^\sigma(t) = 1$. From (3.2) and (3.7), we have

$$\Psi^\sigma(v_\varsigma(t)) \equiv [T(v_1^\sigma(t)) \quad T(v_2^\sigma(t)) \quad \cdots \quad T(v_{J^\sigma}^\sigma(t))]^T$$

$$= [(\sum_{\varphi_1=0}^{1} h_{1\varphi_1}^\sigma(t) g_{1\varphi_1}^\sigma) v_1^\sigma(t) \ \ (\sum_{\varphi_2=0}^{1} h_{2\varphi_2}^\sigma(t) g_{2\varphi_2}^\sigma) v_2^\sigma(t) \ \ \cdots \ \ (\sum_{\varphi_J=0}^{1} h_{J^\sigma\varphi_J}^\sigma(t) g_{K^\sigma\varphi_J}^\sigma) v_{J^\sigma}^\sigma(t)]^T \tag{3.8}$$

Therefore, the final output of the NN model (3.3) can be reformulated as follows:

$$\begin{aligned} \dot{X}(t) &= \sum_{p=0}^{1} h_{\varsigma p}^S(t) G^S (W^S [\cdots [\sum_{n=0}^{1} h_{\varsigma n}^2(t) G^2 (W^2 [\sum_{b=0}^{1} h_{\varsigma b}^1(t) G^1 (W^1 \Lambda(t))])] \cdots]) \\ &= \sum_{p=0}^{1} \cdots \sum_{n=0}^{1} \sum_{b=0}^{1} h_{\varsigma p}^S(t) \cdots h_{\varsigma n}^2(t) h_{\varsigma b}^1(t) G^S W^S \cdots G^2 W^2 G^1 W^1 \Lambda(t) \\ &= \sum_{\Omega} h_{\varsigma\Omega}^\sigma(t) C_\Omega^\sigma \ \Lambda(t) \end{aligned} \tag{3.9}$$

where

$$\sum_{b=0}^{1} h_{\varsigma b}^1(t) \equiv \sum_{b_1=0}^{1} h_{1 b_1}^1(t) \sum_{b_2=0}^{1} h_{2 b_2}^1(t) \cdots \sum_{b_J=0}^{1} h_{J^1 b_J}^1(t)$$

$$\sum_{n=0}^{1} h_{\varsigma n}^2(t) \equiv \sum_{n_1=0}^{1} h_{1 n_1}^2(t) \sum_{n_2=0}^{1} h_{2 n_2}^2(t) \cdots \sum_{n_J=0}^{1} h_{J^2 n_J}^2(t)$$

$$\vdots$$

$$\sum_{p=0}^{1} h_{\varsigma p}^S(t) \equiv \sum_{p_1=0}^{1} h_{1 p_1}^S(t) \sum_{p_2=0}^{1} h_{2 p_2}^S(t) \cdots \sum_{p_J=0}^{1} h_{J^S p_J}^S(t)$$

$$\sum_{\Omega} h_{\varsigma\Omega}^\sigma(t) \equiv \sum_{p=0}^{1} \cdots \sum_{n=0}^{1} \sum_{b=0}^{1} h_{\varsigma p}^S(t) \cdots h_{\varsigma n}^2(t) h_{\varsigma b}^1(t), \ \varsigma = 1,\ 2,\ \cdots,\ J^\sigma;$$

$$C_\Omega^\sigma \equiv G^S W^S \cdots G^2 W^2 G^1 W^1$$

and b_ς, n_ς, $p_\varsigma (\varsigma = 1,\ 2, \cdots, J)$ represent the variables ϕ of the ς th neuron of the first, second, and the Sth layer, respectively. Finally, according to (3.4), the dynamics of the NN model (3.9) can be rewritten as the following LDI state-space representation:

$$\dot{X}(t) = \sum_{i=1}^{\phi} h_i(t) C_i \Lambda(t) \tag{3.10}$$

where $h_i(t) \geq 0$, $\sum_{i=1}^{\phi} h_i(t) = 1$, φ is a positive integer and C_i is a constant matrix with appropriate dimension associated with C_Ω^σ. Moreover, the LDI state-space representation (3.10) can be rearranged as follows:

$$\dot{X}(t) = \sum_{i=1}^{\phi} h_i(t) \{A_i X(t) + \sum_{k=1}^{m} \overline{A}_{ik} X(t - \tau_k)\} \tag{3.11}$$

where A_i and $\overline{A}_{ik}$ are the partitions of C_i corresponding to the partitions of $\Lambda^T(t)$.

Remark 3.2.1: The final output of the NN model (3.3) is converted into an "imitational" fuzzy model Eq. (3.11) via the interpolation method and the technique of LDI state-space representation. That is to say, the "imitational" fuzzy model Eq. (3.11) is an NN model in essence.

IV. Exponential Chaotic Synchronization

In this section, the exponential synchronization scheme of the multiple time-delay chaotic systems is described as below.

A. Master-slave system

Consider the modeling of the master and slave chaotic systems described by the following LDI state-space representations (4.1) and (4.2), respectively.

$$N_m : \dot{X}(t) = \sum_{i=1}^{\varphi} h_i(t)\{A_i X(t) + \sum_{k=1}^{m} \overline{A}_{ik} X(t-\tau_k)\} + D(t) \tag{4.1}$$

$$N_s : \dot{\hat{X}}(t) = \sum_{j=1}^{\varphi} \hat{h}_j(t)[\hat{A}_j \hat{X}(t) + \sum_{k=1}^{m} \hat{\overline{A}}_{jk} \hat{X}(t-\tau_k)] + BU(t) \tag{4.2}$$

Remark 4.1.1 [38]: Due to the boundedness of the chaotic signals, there exists positive constant χ_{ub} and $\chi_{k_{ub}}$ such that $\|\hat{X}(t)\| \le \chi_{ub}$ and $\|\hat{X}(t-\tau_k)\| \le \chi_{k_{ub}}$

The error vector is defined as $E(t) \equiv X(t) - \hat{X}(t) = [e_1(t),\ e_2(t),\ \cdots, e_{\varphi}(t)]^T$. Hence, the dynamics of the error system is described as follows:

$$\begin{aligned} N_e : \dot{E}(t) &= \dot{X}(t) - \dot{\hat{X}}(t) = \sum_{i=1}^{\varphi} h_i(t)\,[A_i X(t) + \sum_{k=1}^{m} \overline{A}_{ik} X(t-\tau_k)] + D(t) \\ &\quad - \sum_{j=1}^{\varphi} \hat{h}_j(t)\,[\hat{A}_j \hat{X}(t) + \sum_{k=1}^{m} \hat{\overline{A}}_{jk} \hat{X}(t-\tau_k)] - BU(t) \\ &= \sum_{i=1}^{\varphi}\sum_{j=1}^{\varphi} h_i(t)\hat{h}_j(t)\,[A_i E(t) + (A_i - \hat{A}_j)\hat{X}(t) + \sum_{k=1}^{m}(\overline{A}_{ik} - \hat{\overline{A}}_{jk})\hat{X}(t-\tau_k) \\ &\quad + \sum_{k=1}^{m} \overline{A}_{ik} E(t-\tau_k) + B\sum_{l=1}^{\rho} \overline{h}_l(t) F_l E(t) + D(t)] \\ &= \sum_{i=1}^{\varphi}\sum_{j=1}^{\varphi}\sum_{k=1}^{m}\sum_{l=1}^{\rho} h_i(t)\hat{h}_j(t)\overline{h}_l(t)\,[N_{il}E(t) + (A_i - \hat{A}_j)\hat{X}(t) + (\overline{A}_{ik} - \hat{\overline{A}}_{jk})\hat{X}(t-\tau_k) \\ &\quad + \overline{A}_{ik} E(t-\tau_k) + D(t)] \end{aligned} \tag{4.3}$$

with $N_{il} \equiv A_i + BF_l$.

B. Fuzzy Controller

According to the control scheme, a fuzzy controller is utilized to stabilize the error system.

The fuzzy controller takes the following form:

Control Rule l: IF $e_1(t)$ is M_{l1} and $\cdots$ and $e_{\delta}(t)$ is $M_{l\delta}$

THEN $U(t) = -F_l E(t)$

where $l = 1,\ 2,\ \cdots,\ \rho$, and ρ is the number of IF-THEN rules of the fuzzy controller and $M_{l\eta}(\eta = 1,\ 2,\ \cdots,\ \delta)$ are the fuzzy sets. Hence, the final output of this fuzzy controller is inferred as follows:

$$U(t) = \frac{-\sum_{l=1}^{\rho} w_l(t) F_l E(t)}{\sum_{l=1}^{\rho} w_l(t)} = -\sum_{l=1}^{\rho} \overline{h}_l(t) F_l E(t) \tag{4.2.1}$$

with $w_l(t) \equiv \prod_{\eta=1}^{\delta} M_{l\eta}(e_{\eta}(t))$, $\overline{h}_l(t) \equiv \frac{w_l(t)}{\sum_{l=1}^{\rho} w_l(t)}$, in which $M_{l\eta}(e_{\eta}(t))$ is the grade of membership of $e_{\eta}(t)$ in $M_{l\eta}$. In this study, it is also assumed that $w_l(t) \ge 0\ (l = 1,\ 2,\ \cdots,\ \rho)$ and $\sum_{l=1}^{\rho} w_l(t) > 0$ for all t. Therefore, $\overline{h}_l(t) \ge 0$ and $\sum_{l=1}^{\rho} \overline{h}_l(t) = 1$ for all t.

C. Delay-Dependant Stability Criterion for Exponential Synchronization

In the following, a delay-dependant criterion is proposed to guarantee the stability of the error system described in (4.3). Prior to examination of stability of the error system, some useful concepts are given below.

Lemma 1 [39]: For real matrices A and B with appropriate dimensions, we have

$$A^T B + B^T A \le \lambda A^T A + \lambda^{-1} B^T B$$

where λ is a positive constant.

Definition 1 [40]**:** The slave system (4.2) can exponentially synchronize with the master system (4.1) (i.e., the error system (4.3) is exponentially stable) if there exist two positive numbers α and β such that the synchronous error satisfies:

$$\|E(t)\| \le \alpha\ exp(-\beta(t - t_0)),\quad \forall\ t \ge 0$$

The positive number β is called the exponential convergence rate.

Theorem 1: The slave system (4.2) can exponentially synchronize with the master system (4.1) (i.e., the error system (4.3) is exponentially stable) in the region specified by $\|E(t)\| \ge \gamma$ and $\|E(t-\tau_k)\| \ge \overline{\gamma}_k$ (γ and $\overline{\gamma}_k$ are positive constants) if there exist symmetric positive definite matrices $\boldsymbol{P}$, ψ_k and positive constants $a,\ b,\ c\ ,d\ ,\ n$ such that the following inequalities hold:

$$\Delta_{ijl} \equiv nN_{il}^{\mathrm{T}} N_{il} + \sum_{k=1}^{m} \tau_k^2 P^2 (a^{-1}m + b^{-1} + c^{-1}m + d^{-1}\mathrm{m} + n^{-1}) + \sum_{k=1}^{m} \psi_k + \varepsilon I < 0 \tag{4.4a}$$

$$\nabla_{ijk} \equiv am\overline{A}_{ik}^T \overline{A}_{ik} - \psi_k + \varepsilon_k cmI < 0 \tag{4.4b}$$

for $i = 1,\ 2,\ \cdots,\ \varphi$; $j = 1,\ 2,\ \cdots,\ \varphi$; $k = 1,\ 2,\ \cdots,\ m$ and $l = 1,\ 2,\ \cdots,\ \rho$.

where

$\varepsilon \equiv \gamma^{-2}\left(bm\chi_{ub}^2\lambda_{\max}[(A_i-\hat{A}_j)^T(A_i-\hat{A}_j)]+dmD_{ub}^{\ 2}\right)$,

$\varepsilon_k \equiv \dfrac{\chi_{k_{ub}}^2}{\overline{\gamma}_k^2}\lambda_{\max}[(A_{ik}-\hat{A}_{jk})^T(A_{ik}-\hat{A}_{jk})]$, and

$N_{il} \equiv A_i - BF_l$.

Remark 4.3.1: Since the matrices Δ_{ijl} must be negative definite to meet the stability condition (4.4a), the larger delay τ_k will make **Theorem 1** more difficult to be satisfied.

Remark 4.3.2: As assumed in **Theorem 1**, γ and $\overline{\gamma}_k$ are arbitrary parameters which can be chosen by the designer. The smaller γ and $\overline{\gamma}_k$, the tighter the error bounds and hence, the better the synchronization performance. However, the smaller γ and $\overline{\gamma}_k$ will result in the larger ε and ε_k, and then make the stability conditions (4.4a) and (4.4b) more difficult to be satisfied.

Remark 4.3.3: In addition, the fuzzy controller which meets the stability conditions (4.4a) and (4.4b) can guarantee exponential convergence and stability in the region specified by $\|E(t)\| \geq \gamma$ and $\|E(t-\tau_k)\| \geq \overline{\gamma}_k$.

Remark 4.3.4: Eqs. (4.4a) and (4.4b) can be reformulated into LMI via the following procedure. By introducing the new variables $Q=P^{-1}$, $K_l=F_lQ$ and $\overline{\psi}_k = Q\psi_k Q$, Eq. (4.4a) can be (is then) rewritten as follows:

$$QnA_i^TA_iQ - QnA_i^TBK_l - nK_l^TBA_iQ + K_l^TnB^2K_l$$
$$+\sum_{k=1}^{m}\tau_k^2(a^{-1}m+b^{-1}+c^{-1}m+d^{-1}m+n^{-1})+\sum_{k=1}^{m}\overline{\psi}_k+Q\varepsilon Q<0 \quad (4.5)$$

Based (Furthermore,) on Schur's complement [37], it is easy to show (find) that the linear matrix inequalities in Eqs. (4.5) and (4.4b) are equivalent to the following LMI in Eqs. (4.6a) and (4.6b):

$$\begin{bmatrix} \Xi & I \\ I & -I \end{bmatrix} < 0 \quad (4.6a)$$

$$\begin{bmatrix} -a^{-1}\overline{\psi}_k m^{-1} + ca^{-1}Q\varepsilon_k Q & Q\overline{A}_{ik}^T \\ \overline{A}_{ik}Q & -I \end{bmatrix} < 0$$
(4.6b)

where

$$\Xi \equiv QnA_i^TA_iQ - QnA_i^TBK_l - nK_l^TBA_iQ + K_l^TnB^2K_l$$
$$+\sum_{k=1}^{m}\tau_k^2(a^{-1}m+b^{-1}+c^{-1}m+d^{-1}m+n^{-1})+\sum_{k=1}^{m}\overline{\psi}_k+Q\varepsilon Q - I .$$

Therefore, **Theorem 1** can be transformed into an LMI problem and efficient interior-point algorithms are now available in Matlab LMI Solver to solve this problem.

Remark 4.3.5 [41]: In order verify the feasibility of solving the inequalities Eqs. (4.6a, 4.6b) by LMI Solver (Matlab), the interior-point optimization techniques are utilized to compute feasible solutions. Such techniques require that the system of LMI constraints be strictly feasible, that is, the feasible set has a nonempty interior. For feasibility problems, the LMI Solver by *feasp* ♣ is shown as follows:

Find x such that the LMI $L(x)<0$ ♦ (4.7a)

as

Minimize t subject to $L(x) < t \times I$. (4.7b)

From the above, the LMI constraint is always strictly feasible in x, t and the original LMI (4.7a) is feasible if and only if the global minimum $t\min$ of (4.7b) satisfies $t\min < 0$. In other words, if $t\min < 0$ will make the stability conditions Eqs. (4.4a) and (4.4b) in **Theorem 1** can be met. Based on **Theorem 1**, a fuzzy controller can be synthesized to exponentially stabilize the error system.

Remark 4.3.6: In order reduce the computational burden, this study sets the positive constants *a*, *b*, *c* and *n* are chosen to be unity in this study.

V. NUMERICAL EXAMPLE

The following example is given to illustrates the effectiveness of the algorithm presented above.

Problem: The purpose of this example is to synthesize a fuzzy controller to realize the exponential synchronization with the error bounds $\gamma=0.2$, $\overline{\gamma}_1=0.2$, $\overline{\gamma}_2=0.2$ for the modified multiple time-delay Chua's oscillator circuit described as follows:

$$\begin{bmatrix}\dot{v}_{C_1}(t)\\ \dot{v}_{C_2}(t)\\ \dot{i}_L(t)\end{bmatrix}=\begin{bmatrix}\frac{-1}{C_1R} & 0 & \frac{1}{C_1R} & 0 & 0\\ 0 & \frac{1}{C_2R} & 0 & \frac{-1}{C_2R} & \frac{1}{C_2}\\ 0 & 0 & \frac{-1}{L} & 0 & 0\end{bmatrix}\begin{bmatrix}v_{C_1}(t)\\ v_{C_1}(t-0.05)\\ v_{C_2}(t)\\ v_{C_2}(t-0.1)\\ i_L(t)\end{bmatrix}+\begin{bmatrix}\frac{-\Im(v_{C_1})}{C_1}\\ 0\\ 0\end{bmatrix}$$
(5.1)

with $R=1.558$, $C_1=0.1$, $C_2=2$, L=1/7, where $[v_{C_1}(t)\ v_{C_2}(t)\ i_L(t)]^T$ is the state vector and the initial condition is (-3, 1, -1). Moreover, the upper bounds of the chaotic signals of the slave system are $\chi_{ub}=37$, $\chi_{1_{ub}}=37$, $\chi_{2_{ub}}=37$ and the transmitted noise $D(t)=[0.5\sin(1.581t)\quad 0\quad 0]^T$. The nonlinear function $\Im(v_{C_1}(t))$ characterizes the voltage-controlled resistor with a piecewise-linear characteristic (see Fig. 3)

$$\Im(v_{C_1}(t)) = R_b v_{C_1}(t) + \frac{1}{2}(R_a - R_b)\left(\left|v_{C_1}(t)+Z\right| - \left|v_{C_1}(t)-Z\right|\right)$$
(5.2)

♣ *feasp* is the syntax used to test feasibility of a system of LMIs in MATLAB.

♦ In this study, Eq. (4.7a) can be represented as Eq. (4.6a, 4.6b).

$$= \begin{cases} R_b v_{C_1}(t) + (R_a - R_b)Z, & v_{C_1}(t) \ge Z \\ R_a v_{C_1}(t), & -Z < v_{C_1}(t) < Z \\ R_b v_{C_1}(t) - (R_a - R_b)Z, & v_{C_1}(t) \le -Z. \end{cases} \quad (5.3)$$

Notably, the nonlinear term $\ni(v_{C_1}(t))$ only with $R_a \neq R_b$ is of interest (otherwise the modified Chua"s oscillator circuit becomes a simple linear system).

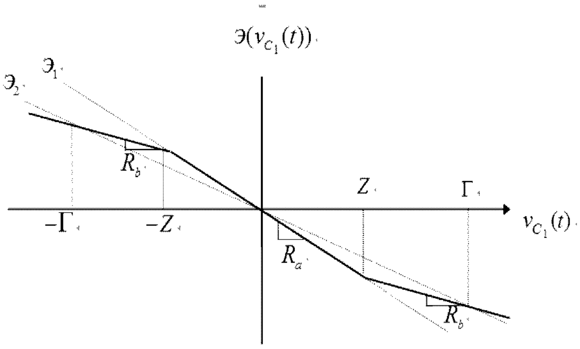

Fig. 3 Resistor characteristic of the modified Chua"s oscillator circuit with $R_a = -4$, $R_b = -0.1$, $Z=1$ and $\Gamma=15$.

Assuming $v_{C_1}(t) \in [-\Gamma, \Gamma]$, $\Gamma > Z > 0$, we obtain the following sector to bound $\ni(v_{C_1}(t))$ (Fig. 3):

$$\ni_1(v_{C_1}(t)) = R_a v_{C_1}(t), \quad (5.4)$$

$$\ni_2(v_{C_1}(t)) = \left(R_b + \frac{(R_a - R_b)Z}{\Gamma} \right) v_{C_1}(t) = R v_{C_1}(t), \quad (5.5)$$

where $R = R_b + \left((R_a - R_b)Z / \Gamma\right)$.

Letting $x = [x_1(t),\ x_2(t),\ x_3(t)]^T = [v_{C_1}(t),\ v_{C_2}(t),\ i_L(t)]^T$, Eq. (5.1) can be rewritten as follows:

$$N_d : \begin{cases} \dot{x}_1(t) = -\frac{1}{C_1 R} x_1(t) - \frac{1}{C_1} \ni(x_1(t)) + \frac{1}{C_1 R} x_2(t) \\ \dot{x}_2(t) = \frac{1}{C_2 R} x_1(t-0.05) - \frac{1}{C_2 R} x_2(t-0.1) + \frac{1}{C_2} x_3(t) \\ \dot{x}_3(t) = -\frac{1}{L} x_2(t) \end{cases} \quad (5.6)$$

The uncontrolled modified Chua"s circuit exhibits a chaotic attractor (as shown in Fig. 4); specifically the so-called double scroll attractor.

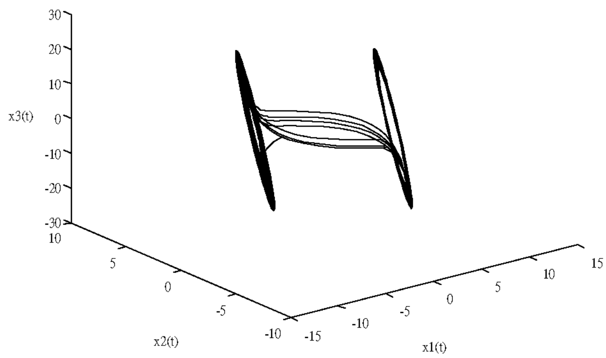

Fig. 4 Phase portrait of $x_1(t)$, $x_2(t)$ and $x_3(t)$.

Solution: We can solve the above problem according to the following steps.

Step 1: Establish an NN model for the chaotic system N_d via back propagation algorithm. The NN model to approximate the chaotic system N_d is constructed by 9–4–3, shown in Fig. 5.

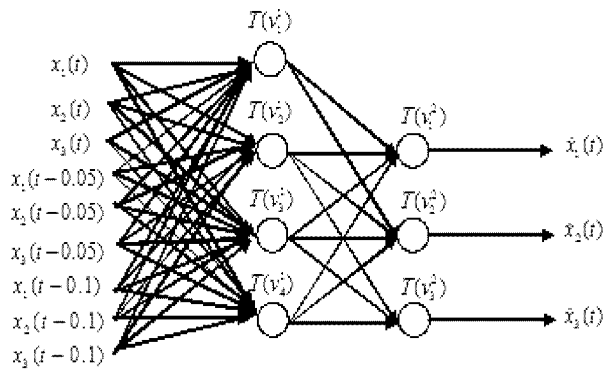

Fig. 5 The NN model of the chaotic system N_d.

The transfer functions of the hidden neurons are chosen as follows:

$$T(v_\varsigma^1(t)) = \{\frac{2}{[1+\exp(-v_\varsigma^1(t)/0.5)]} - 1\}, \text{ for } \varsigma = 1, 2, 3. \quad (5.7a)$$

The transfer functions of the output neurons are chosen as follows:

$$T(v_\varsigma^2(t)) = v_\varsigma^2(t), \text{ for } \varsigma = 1, 2, 3. \quad (5.7b)$$

After training, we can obtain the following the connection weights*

---

* The indices in $W_{\varsigma\vartheta}^{\sigma}$ say that the weight of the σth layer in the NN model represents the connection to the ςth neuron from the ϑth source.

$$\begin{bmatrix} W^1_{11} & W^1_{12} & W^1_{13} & W^1_{14} & W^1_{15} & W^1_{16} & W^1_{17} & W^1_{18} & W^1_{19} \\ W^1_{21} & W^1_{22} & W^1_{23} & W^1_{24} & W^1_{25} & W^1_{26} & W^1_{27} & W^1_{28} & W^1_{29} \\ W^1_{31} & W^1_{32} & W^1_{33} & W^1_{34} & W^1_{35} & W^1_{36} & W^1_{37} & W^1_{38} & W^1_{39} \\ W^1_{41} & W^1_{42} & W^1_{43} & W^1_{44} & W^1_{45} & W^1_{46} & W^1_{47} & W^1_{48} & W^1_{49} \end{bmatrix} \quad (5.8a)$$

$$\begin{bmatrix} W^2_{11} & W^2_{12} & W^2_{13} & W^2_{14} \\ W^2_{21} & W^2_{22} & W^2_{23} & W^2_{24} \\ W^2_{31} & W^2_{32} & W^2_{33} & W^2_{34} \end{bmatrix} \quad (5.8b)$$

From Fig. 5, we have‡:

$$v^1_\varsigma(t) = W^1_{\varsigma 1}x_1(t) + W^1_{\varsigma 2}x_2(t) + W^1_{\varsigma 3}x_3(t) + W^1_{\varsigma 4}x_1(t-0.05)$$
$$+W^1_{\varsigma 5}x_2(t-0.05) + W^1_{\varsigma 6}x_3(t-0.05)$$
$$+W^1_{\varsigma 7}x_1(t-0.1) + W^1_{\varsigma 8}x_2(t-0.1) + W^1_{\varsigma 9}x_3(t-0.1), \ \varsigma = 1,\ 2,\ 3,\ 4 \quad (5.9a)$$

$$v^2_\varsigma(t) = W^2_{\varsigma 1}T(v^1_1(t)) + W^2_{\varsigma 2}T(v^1_2(t)) + W^2_{\varsigma 3}T(v^1_3(t)),$$
$$\varsigma = 1, 2, 3, \quad (5.9b)$$

$$\dot{X}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} T(v^2_1(t)) \\ T(v^2_2(t)) \\ T(v^2_3(t)) \end{bmatrix}. \quad (5.10)$$

According to (3.6), the minimum and the maximum of the derivative of each transfer function shown in (5.7a, 5.7b) can be obtained as follows:

$g^1_{\varsigma 0} = g^2_{\varsigma 0} = 0$, $g^1_{\varsigma 1} = g^2_{\varsigma 1} = 1$, for $\varsigma = 1,\ 2,\ 3.$ (5.11)

To simplify the notation, we let $g^1_{\varsigma 0} = g^1_0$, $g^1_{\varsigma 1} = g^1_1$, $g^2_{\varsigma 0} = g^2_0$, and $g^2_{\varsigma 1} = g^2_1$. Then, based on the interpolation method, we have

$$\dot{x}_1(t) = \sum_{c=0}^{1} h^2_{1c}(t) g^2_c \sum_{s=0}^{1}\sum_{p=0}^{1}\sum_{r=0}^{1} h^1_{1s}(t) h^1_{2p}(t) h^1_{3r}(t) (g^1_s W^2_{11} v^1_1(t)$$
$$+ g^1_p W^2_{12} v^1_2(t) + g^1_r W^2_{13} v^1_3(t)), \quad (5.12)$$

$$\dot{x}_2(t) = \sum_{a=0}^{1} h^2_{2a}(t) g^2_a \sum_{s=0}^{1}\sum_{p=0}^{1}\sum_{r=0}^{1} h^1_{1s}(t) h^1_{2p}(t) h^1_{3r}(t) (g^1_s W^2_{21} v^1_1(t)$$
$$+ g^1_p W^2_{22} v^1_2(t) + g^1_r W^2_{23} v^1_3(t)), \quad (5.13)$$

$$\dot{x}_3(t) = \sum_{d=0}^{1} h^2_{3d}(t) g^2_d \sum_{s=0}^{1}\sum_{p=0}^{1}\sum_{r=0}^{1} h^1_{1s}(t) h^1_{2p}(t) h^1_{3r}(t) (g^1_s W^2_{31} v^1_1(t)$$
$$+ g^1_p W^2_{32} v^1_2(t) + g^1_r W^2_{33} v^1_3(t)), \quad (5.14)$$

‡ The symbol v^σ_ς denotes the net input of the ςth neuron of the σ th layer in the NN model, and the indices σ and ς shown in $h^\sigma_{\varsigma\varphi}$ ($\varphi = 1,\ 2$) indicate the same thing.

By plugging Eqs. (5.9a, 5.9b) into Eqs. (5.12, 5.13, 5.14), we obtain

$$\dot{X}(t) = \sum_{c=0}^{1}\sum_{a=0}^{1}\sum_{d=0}^{1}\sum_{s=0}^{1}\sum_{p=0}^{1}\sum_{r=0}^{1} h^2_{1c}(t) h^2_{2a}(t) h^2_{3d}(t) h^1_{1s}(t) h^1_{2p}(t) h^1_{3r}(t)$$
$$\square \{A_{cadspr} X(t) + \bar{A}_{cadspr1} X(t-0.05) + \bar{A}_{cadspr2} X(t-0.1)\} \quad (5.15)$$

where

$$A_{cadspr} = \begin{bmatrix} g^2_c(g^1_s W^2_{11} W^1_{11} + g^1_p W^2_{12} W^1_{21} + g^1_r W^2_{13} W^1_{31}) & g^2_c(g^1_s W^2_{11} W^1_{12} + g^1_p W^2_{12} W^1_{22} + g^1_r W^2_{13} W^1_{32}) & g^2_c(g^1_s W^2_{11} W^1_{13} + g^1_p W^2_{12} W^1_{23} + g^1_r W^2_{13} W^1_{33}) \\ g^2_a(g^1_s W^2_{21} W^1_{11} + g^1_p W^2_{22} W^1_{21} + g^1_r W^2_{23} W^1_{31}) & g^2_a(g^1_s W^2_{21} W^1_{12} + g^1_p W^2_{22} W^1_{22} + g^1_r W^2_{23} W^1_{32}) & g^2_a(g^1_s W^2_{21} W^1_{13} + g^1_p W^2_{22} W^1_{23} + g^1_r W^2_{23} W^1_{33}) \\ g^2_d(g^1_s W^2_{31} W^1_{11} + g^1_p W^2_{32} W^1_{21} + g^1_r W^2_{33} W^1_{31}) & g^2_d(g^1_s W^2_{31} W^1_{12} + g^1_p W^2_{32} W^1_{22} + g^1_r W^2_{33} W^1_{32}) & g^2_d(g^1_s W^2_{31} W^1_{13} + g^1_p W^2_{32} W^1_{23} + g^1_r W^2_{33} W^1_{33}) \end{bmatrix}$$

$$\bar{A}_{cadspr1} = \begin{bmatrix} g^2_c(g^1_s W^2_{11} W^1_{14} + g^1_p W^2_{12} W^1_{24} + g^1_r W^2_{13} W^1_{34}) & g^2_c(g^1_s W^2_{11} W^1_{15} + g^1_p W^2_{12} W^1_{25} + g^1_r W^2_{13} W^1_{35}) & g^2_c(g^1_s W^2_{11} W^1_{16} + g^1_p W^2_{12} W^1_{26} + g^1_r W^2_{13} W^1_{36}) \\ g^2_a(g^1_s W^2_{21} W^1_{14} + g^1_p W^2_{22} W^1_{24} + g^1_r W^2_{23} W^1_{34}) & g^2_a(g^1_s W^2_{21} W^1_{15} + g^1_p W^2_{22} W^1_{25} + g^1_r W^2_{23} W^1_{35}) & g^2_a(g^1_s W^2_{21} W^1_{16} + g^1_p W^2_{22} W^1_{26} + g^1_r W^2_{23} W^1_{36}) \\ g^2_d(g^1_s W^2_{31} W^1_{14} + g^1_p W^2_{32} W^1_{24} + g^1_r W^2_{33} W^1_{34}) & g^2_d(g^1_s W^2_{31} W^1_{15} + g^1_p W^2_{32} W^1_{25} + g^1_r W^2_{33} W^1_{35}) & g^2_d(g^1_s W^2_{31} W^1_{16} + g^1_p W^2_{32} W^1_{26} + g^1_r W^2_{33} W^1_{36}) \end{bmatrix}$$

$$\bar{A}_{cadspr2} = \begin{bmatrix} g^2_c(g^1_s W^2_{11} W^1_{17} + g^1_p W^2_{12} W^1_{27} + g^1_r W^2_{13} W^1_{37}) & g^2_c(g^1_s W^2_{11} W^1_{18} + g^1_p W^2_{12} W^1_{28} + g^1_r W^2_{13} W^1_{38}) & g^2_c(g^1_s W^2_{11} W^1_{19} + g^1_p W^2_{12} W^1_{29} + g^1_r W^2_{13} W^1_{39}) \\ g^2_a(g^1_s W^2_{21} W^1_{17} + g^1_p W^2_{22} W^1_{27} + g^1_r W^2_{23} W^1_{37}) & g^2_a(g^1_s W^2_{21} W^1_{18} + g^1_p W^2_{22} W^1_{28} + g^1_r W^2_{23} W^1_{38}) & g^2_a(g^1_s W^2_{21} W^1_{19} + g^1_p W^2_{22} W^1_{29} + g^1_r W^2_{23} W^1_{39}) \\ g^2_d(g^1_s W^2_{31} W^1_{17} + g^1_p W^2_{32} W^1_{27} + g^1_r W^2_{33} W^1_{37}) & g^2_d(g^1_s W^2_{31} W^1_{18} + g^1_p W^2_{32} W^1_{28} + g^1_r W^2_{33} W^1_{38}) & g^2_d(g^1_s W^2_{31} W^1_{19} + g^1_p W^2_{32} W^1_{29} + g^1_r W^2_{33} W^1_{39}) \end{bmatrix}$$

$X(t) = [x_1(t) \ \ x_2(t) \ \ x_3(t)]^T$,
$X(t-0.05) = [x_1(t-0.05) \ \ x_2(t-0.05) \ \ x_3(t-0.05)]^T$ and
$X(t\text{-}0.1) = [x_1(t\text{-}0.1) \ \ x_2(t\text{-}0.1) \ \ x_3(t\text{-}0.1)]^T$.

Next, by renumbering the matrices shown in Eq. (5.15), the NN model can be rewritten as the following LDI state-space representation:

$$\dot{X}(t) = \sum_{i=1}^{128} h_i(t)\{A_i X(t) + \sum_{k=1}^{2} \bar{A}_{ik} X(t-\tau_k)\} \quad (5.16)$$

where $\tau_1 = 0.05$, $\tau_2 = 0.1$,

$A_1 = A_{0000000}$, $A_2 = A_{0000001}$, … , $A_{127} = A_{1111110}$, $A_{128} = A_{1111111}$,

$\bar{A}_{1\,1} = \bar{A}_{00000000\,1}$, $\bar{A}_{256\,1} = \bar{A}_{11111111\,1}$,

$\bar{A}_{1\,2} = \bar{A}_{00000000\,2}$, $\bar{A}_{256\,2} = \bar{A}_{11111111\,2}$,

Step 2: The Eq. (5.16) is adopted for the NN model of the master system and slave system, respectively.

$$\text{Master:} \quad \dot{X}(t) = \sum_{i=1}^{128} h_i(t)\{A_i X(t) + \sum_{k=1}^{2} \bar{A}_{ik} X(t-\tau_k)\} + D(t)$$

(5.17)

$$\text{Slave:} \quad \dot{\hat{X}}(t) = \sum_{j=1}^{128} \hat{h}_j(t)\{\hat{A}_j \hat{X}(t) + \sum_{k=1}^{2} \hat{\bar{A}}_{jk} \hat{X}(t-\tau_k)\} + BU(t)$$

(5.18)

with $\tau_1 = 0.05$, $\tau_2 = 0.1$. Moreover, the initial condition of the master system and slave system are (-3, 1, -1) and (-2.5, 1.5, -0.5), respectively. Therefore, the dynamics of the error system is gotten as follows:

$$\dot{E}(t) = \dot{X}(t) - \dot{\hat{X}}$$
$$= \sum_{i=1}^{128}\sum_{j=1}^{128}\sum_{k=1}^{2} h_i(t)\hat{h}_j(t)\,[(A_i E(t) + (A_i - \hat{A}_j)\hat{X}(t)$$
$$+(\overline{A}_{ik} - \hat{\overline{A}}_{jk})\hat{X}(t-\tau_k) + \overline{A}_{ik}E(t-\tau_k) - BU(t)] + D(t) \tag{5.19}$$

Step 3: According to the interpolation algorithm, the nonlinear term $э(e_1)$ can be described by

$$э(e_1) = M_1(e_1)э_1(e_1) + M_2(e_1)э_2(e_1), \tag{5.20}$$

where $M_1(e_1), M_2(e_1) \in [0, 1]$ and $M_1(e_1) + M_2(e_1) = 1$. This implies that the nonlinear term can be interpolated using $э_1(e_1)$ and $э_2(e_1)$. Substituting (5.3)-(5.5) into (5.20), we can derive the following membership functions for each e_1 (see Fig. 6):

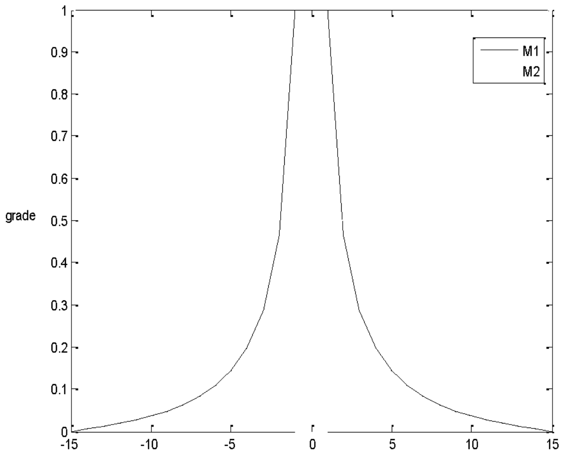

Fig. 6 Membership functions for the modified Chua"s oscillator circuit.

$$M_1(e_1) = \begin{cases} \dfrac{-\dfrac{Z}{\Gamma}e_1 + Z}{(1-\dfrac{Z}{\Gamma})e_1}, & e_1 \geq Z \\ 1, & -Z < e_1 < Z, \\ \dfrac{-\dfrac{Z}{\Gamma}e_1 - Z}{(1-\dfrac{Z}{\Gamma})e_1}, & e_1 \leq -Z \end{cases} \tag{5.21}$$

$$M_2(e_1) = 1 - M_1(e_1). \tag{5.22}$$

In order to achieve the synchronization, a fuzzy controller is synthesized as follows:

Control Rule 1: IF $e_1(t)$ is M_1 , THEN $U(t) = -F_1E(t)$.

Control Rule 2: IF $e_1(t)$ is M_2 , THEN $U(t) = -F_2E(t)$. (5.23)

According to Eq. (3.12), the overall fuzzy controller is

$$U(t) = -\frac{\sum_{l=1}^{2} w_l(t)F_lE(t)}{\sum_{l=1}^{2} w_l(t)} = -\sum_{l=1}^{2}\overline{h}_l(t)F_lE(t) \tag{5.24}$$

with $w_l(t) \equiv M_l(e_1(t))$, $\overline{h}_l(t) \equiv \dfrac{w_l(t)}{\sum_{l=1}^{2} w_l(t)}$.

In order to meet **Theorem 1**, the matrices Δ_{ijl}"s and ∇_{ijk} „s described in Eq. (4.4a, 4.4b) must be negative definite. At first, based on Eqs. (5.8, 5.17-5.19, 5.23-5.24) and Eq. (4.4a, 4.4b), we can get the common solutions P , ψ_1 and ψ_2 via Matlab LMI toolbox with *a*=1, *b*=1, *c*=1, d=1 and *n*=1 for reducing the computational burden:

$$P = 1.0\text{e+}005 * \begin{bmatrix} 5.5205 & -0.0000 & 0.0000 \\ -0.0000 & 5.5205 & -0.0000 \\ 0.0000 & -0.0000 & 5.5205 \end{bmatrix}, \tag{5.25}$$

$$\psi_1 = 1.0\text{e+}016 * \begin{bmatrix} 1.6712 & -0.0000 & 0.0000 \\ -0.0000 & 1.6712 & -0.0000 \\ 0.0000 & -0.0000 & 1.6712 \end{bmatrix} \tag{5.26}$$

$$\psi_2 = 1.0\text{e+}016 * \begin{bmatrix} 1.6712 & -0.0000 & 0.0000 \\ -0.0000 & 1.6712 & -0.0000 \\ 0.0000 & -0.0000 & 1.6712 \end{bmatrix} \tag{5.27}$$

Furthermore, the resulting controller gains are

$$F_1 = 10^{10} * \begin{bmatrix} 0.2055 & -0.0080 & 0.0269 \\ 0.6579 & -2.1920 & 7.6699 \\ 0.0939 & -0.5748 & -2.6488 \end{bmatrix}, \tag{5.28}$$

$$F_2 = 10^{10} * \begin{bmatrix} 0.2098 & -0.0074 & 0.0166 \\ 0.5760 & -2.0090 & 4.6861 \\ 0.0471 & -0.3512 & -2.2875 \end{bmatrix}. \tag{5.29}$$

and the best value $tmin$ of LMI Solver (Matlab) is -1.414637e-006. According to **Remark 4.3.5**, the error system (5.19) can be exponentially stabilized. Obviously, the exponential synchronization can be realized by the designed fuzzy controller (see Fig. 7). Figs. 8a-8c illustrate that the synchronization errors $(e_1, e_2\, and\, e_3)$. Moreover, the assumptions of error bounds and upper bounds of the chaotic signals of the slave system given in Problem are satisfied from the illustration shown in Fig. 9 and Fig. 10a-10b, respectively.

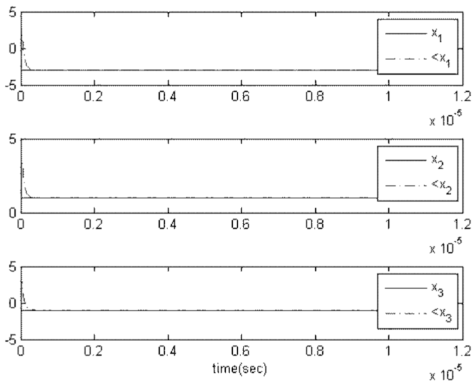

Fig. 7. State responses of both master and slave systems.

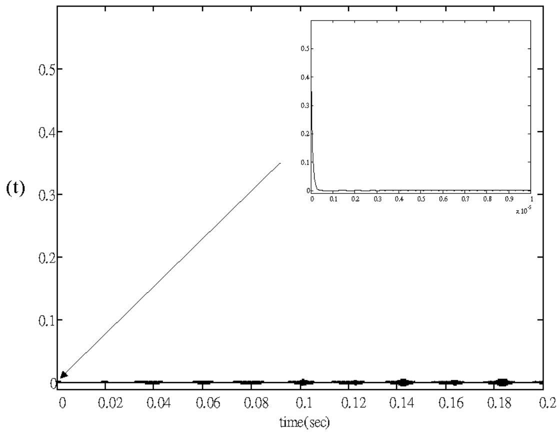

Fig. 8a Time responses of synchronization error $e_1(t)$
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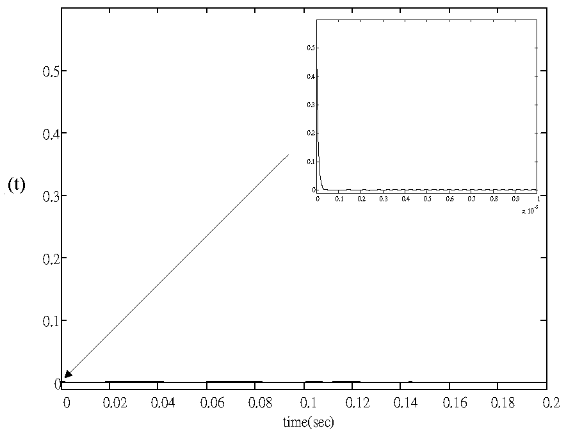

Fig. 8b Time responses of synchronization error $e_2(t)$.

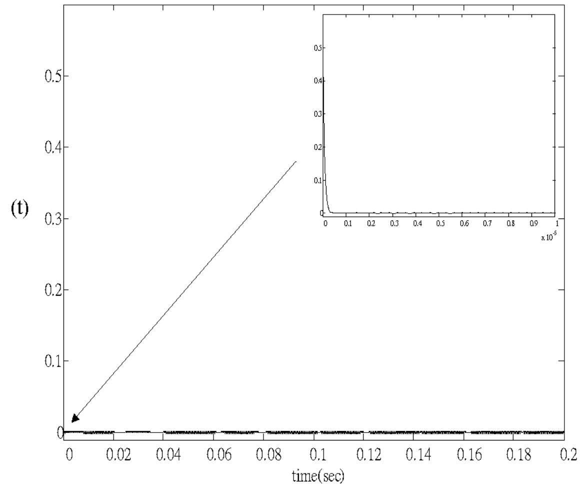

Fig. 8c Time responses of synchronization error $e_3(t)$.

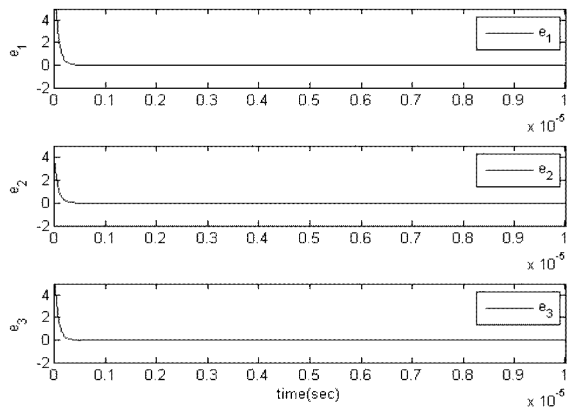

Fig. 9.State error responses of both master and slave systems.

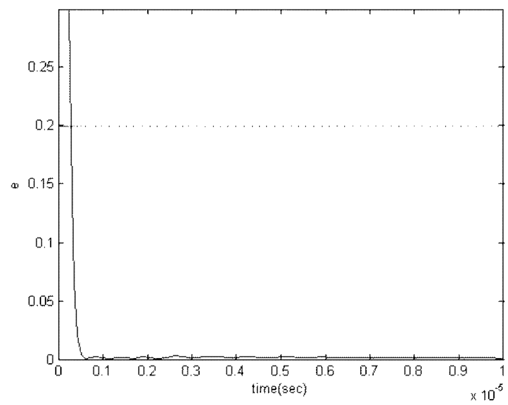

Fig. 10a The E-norm of slave system with the controller.

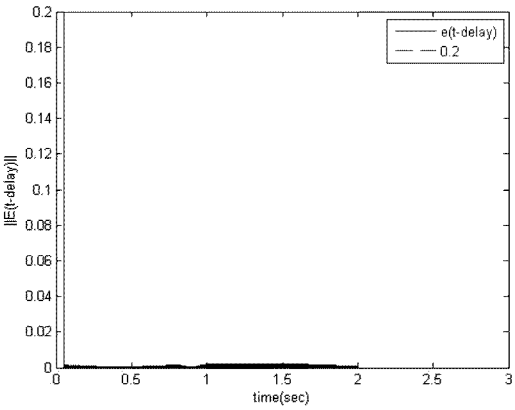

Fig. 10b The E-delay norm of slave system with the controller.

VI. CONCLUSION

This study presents an effective approach to realize the exponential synchronization of multiple time delays chaotic systems. First, the neural-network (NN) model is employed to approximate the chaotic systems with multiple time delays. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of the NN model. Next, in terms of Lyapunov‟s direct method, a delay-dependant criterion is derived to guarantee the stability of the error system between the master system and slave system. Subsequently, the stability condition of this criterion is reformulated into a linear matrix inequality (LMI). Based on the LMI, a fuzzy controller is synthesized to realize the exponential synchronization of the chaotic master-slave systems. Finally, simulation results demonstrate that the exponential synchronization of multiple time-delay chaotic systems can be achieved by the fuzzy controller designed via the LDI state-space representation.

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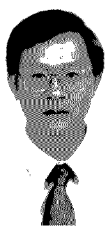
VIII. BIOGRAPHIES



Feng-Hsiag Hsiao was born in Tainan, Taiwan, R.O.C., in 1960. He received the B.S. degree in electronic engineering from Chung Yuan Christian University, Chung-Li, Taiwan, in 1983, the M.S. degree in electrical engineering from Tatung University, Taipei, Taiwan, in 1985, and the Ph.D. degree in electrical engineering from National Sun Yat-Sen University, Kaohsiung, Taiwan, in 1991. He is currently a Full Professor with the Department of Electrical Engineering, National University of Tainan, Tainan, Taiwan. His research interests are in the area of fuzzy control, neural network, large-scale control, and the dither problem.